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ABSTRACT

The main objective of this document is to give an outline of the reform movement within mathematics education in the United States from 1951 until the late 1970's. The reform movement is divided into three phases which are distinguished by certain features common to events of the period. The material is divided into the following main chapter sections: (1) Causes of the Reform Movement; (2) The Beginning of Curriculum Reform (1951-1962); (3) The Zenith of the Curriculum Reform (1963-1971); (4) Decline of the New Math; and (5) General Survey of the Entire Reform. Descriptions are given of the main projects, conferences, and other selected events (such as the "Sputnik shock"), various acts related to education, and reaction to the "new mathematics." New goals were set and new methods of teaching mathematics were developed during the period. The movement should be viewed as an evolution rather than a revolution. The study closes with some thoughts regarding future directions for mathematics education. (MP)

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THE NEW MATH

An historical account of the reform of mathematics instruction in the United States of America

by

Elisabeth Barlage

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"Modern Math"

A lot of changes have been made
Since first the world began.
But changes that effect our thought
Have all been made by man.
Yet change must ever plague our mind
Lest dull should be our path
Our feet must find, so I'm resigned
To learn this "modern math".

My child came home from school one day
And brought with her a book.
I picked it up from where it lay
To have a casual look.
Before two pages I had read
I turned a paler shade.
I gazed in awe at what I saw,
Some changes had been made!

The pages pictured many things
Like cows, and ships, and sticks -
And circles, crosses, dots and rings.
Was this arithmetic?
"So George's mother put some salt
Upon her grocery list!"
While George, it seems put salt on beans.
Somehow, the point I missed.

And then the thought occurred to me,
Here's something I should know.
I'll "do" the form, I'll pay the fee,
And back to school I'll go.
So I sat down with other folks
And listened close to Ron
And Sarah, too. The next I knew,
Two pleasant hours had gone.

I never really paid much mind
That six was less than eight,
But now we have a special sign
This point to illustrate.
In fact the symbols we now see
Outdo the alphabet.
There's two or three that baffle me,
And some I'll never get.

There's such a thing as change of base
Just like they have on Mars.
One base of two we use more space
Then if we counted stars.
That there's an art in taking sticks
We learned a lesson, too.
By subtle tricks, I stop on six
And force the last on you!

And then I learned in one short hour
That half of "eight" is three
And "zero" has a special power;
"One" gives identity.
And rules we always thought we knew
Or somehow took for granted
Now need a clue to show what's true:
Signs round, and sharp, and slanted.

No longer do we have quartettes,
And gone are gangs and clubs.
They now are elements of sets,
And all of these have subs.
We have no further need for teams
Or squads or pairs or herds,
And flocks are dreams, for now it seems
That sets are for the birds.

You, too, are member of a set
And no doubt intersect,
And form a union, too, I'll bet,
In ways you don't suspect.
I'm in a bracket now with you
Whom ne'er I knew before.
I'm less than you, or equal to,
Or maybe even more.

I'm going to do my level best
To give my brain a bath;
I'll try to keep up with the rest
And learn this "modern math".
I'll try to get into the mood,
But I'm afraid that yet
I'll sit and brood - and then conclude
My head's the empty set! (Cress (13) p. 139)

Introduction

The poem quoted above directly identifies the subject of this paper, the "New" or "Modern Math". As one can see from the poem the "New Math" involves innovations in school mathematics. These innovations are difficult for parents because mathematics education today is different in many ways from mathematics education of their day. The catchword which has come to epitomize this movement is "set theory", but this expression covers only a small fraction of the innovations and distorts a true picture of the facts and circumstances of the "New Math".

What is meant by "New Math"? "New Math" is a widespread term that refers to that part of the reform movement in the teaching of school mathematics which lasted about two decades beginning approximately 1951. This term was coined in the USA at the beginning of the fifties as scientists began to adjust mathematics education to the new demands of society. At the same time similar efforts arose in other countries. In the course of these endeavors the term "New Math" acquired its international meaning.

The reform movement implies more than just changes of the content of the traditional mathematics syllabus as is the case of set theory. Besides the improvement and updating of traditional syllabi, new goals were set for mathematics education and new methods of teaching mathematics were developed.

The following definition describes the meaning of "New Math" in the sense given above:

> New Math ... refers to two decades of developments that had a general thrust and direction but sprang from many roots, took many different and even opposing forms, evolved and changed with facets disappearing and new ones arising. (CBMS (17) pp.21f.)

The term is used here "as an historical label for the

phenomenon of the very diversified series of developments that took place in school mathematics."[1] The reader should obtain a better understanding of these developments as he proceeds through this paper.

The main objective of this study is to give an outline of the reform movement within mathematics education in the USA from 1951 until the late seventies. In spite of many difficulties that arise from the differing circumstances and developments in the USA, the main issues of the reform will be presented and studied here.

There are several ways to approach this problem. This paper will start with the actual facts about conferences, projects, laws, movements, etc. A description of the reform issues will be developed on these grounds. However, within the range of this paper it is impossible to present every detail. An exact description of the content and a rationale for the new methods could not be presented to everyone's satisfaction. The main concern is the historical development - a presentation of the causes of the reform movement, its contributing forces and influences, and its resulting changes and consequences. To this end, "aims", "contents" and "methods" were chosen as the three main aspects to be used in describing projects and demonstrating the development of the reform.

In this paper the whole reform movement has been devided into three phases which are distinguished by certain features common to events of the period. These three phases will be designated as the beginning, the zenith, and the decline of "New Math". Whereas chapter one starts with a description of the causes of the movement, each of the following three chapters review one of the three phases. A description will be given of the main projects, conferences, and other pertinent events, such as the "sputnik shock", various acts selected to education, and the reaction to "New Math".

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1) CBMS (17) p.22.

Each of these chapters closes with a summary of the crucial points of the respective phase. In the fifth chapter some main issues are traced as they evolve in the course of time, and the underlying principles of the projects will be examined in order to better understand the changes within the reform movement. In the same chapter the question of whether the reform movement succeeded or failed will be considered as well.

1. CAUSES OF THE REFORM MOVEMENT

1.1. Developments within mathematics

In the course of the last century a revolution in the content and nature of mathematics took place. This revolution was marked by more precise definitions and more exact proofs. Hilbert had developed a formal-axiomatic version of geometry by the turn of the century. He renounced ontological statements about mathematical objects. He built up Euclidean Geometry as a system of axioms that is without contradictions, independent and self-contained. Consequently mathematics developed into a science of formal systems. These systems consist of definitions and axioms, forming the basis for new theories.

Besides this way of thinking (formalism), another way of thinking was developed. The aim of this effort was to look for common features of different domains of mathematics. The result was new classifications within mathematics. Relations between sets and elements of sets were examined; common features and relations or operations between elements were investigated. Structures were discovered which served as a new way of classifying mathematics according to an axiomatic approach. The work of classifying mathematics according to these structures was done by a group of French mathematicians, called 'Nicholas Bourbaki'. The classical classification disappeared and gave way to a new one which aimed at combining the once independent domains.

To sum up, innovations within mathematics are characterized by

1. die verstärkte Erforschung und Absicherung der Grundlagen der Mathematik mit der Hilfe der mathematischen Logik und der Mengenlehre, die in den letzten hundert Jahren als eigenständige Disziplin hoch entwickelt wurden,

2. die Neuordnung nach mathematischen Strukturen, in die sich die klassischen Gebiete nach ihren immanenten Wesensmerkmalen gliedern. Diese Neuordnung ermöglicht einen Überblick über das Gesamtgebäude der Mathematik und bewirkt eine beträchtliche Ökonomie des Denkens,
3. die Entwicklung neuer Teilgebiete wie Schaltalgebra, Informatik, Statistik usw.
(Ellrott/Schindler (25)p.23)

Today mathematics is regarded as

die unter Verwendung von Mengenlehre und axiomatischer Methode von Strukturbegriffen neu aufgefaßte und (in Verbindung mit Anwendungsproblemen der verschiedensten Art) weiterzuentwickelnde Mathematik. (Steiner (85) p.14)

1.2. Increasing Role of Mathematics in American Society

In the middle of this century one could have talked of a "scientific revolution" following an "industrial revolution" in the last century. Other expressions as "atomic age" came up. All these labels indicate the increasing influence of science and technology. Together with an increased importance of these disciplines the importance of mathematics has been growing. The discipline of mathematics underwent fundamental changes. Even more important is the growth of its applications. Mathematics is not only applied to natural sciences but also to psychology, sociology, linguistics, philosophy and other disciplines. With the increase of mathematical applications came an increased demand for mathematically trained manpower. American industry needed more and more specialists such as engineers, technicians, and scientists. The government needed mathematically trained personnel, especially in military service and research.[1] In summary it can be said that the importance of mathematics and with it the need for mathematically trained specialists increases with the increasing influence of science and technology.[2]

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1) see Lloyd (46)
2) see Education Committee of the Industrial Mathematics Society (23)

1.3. Criticism of Traditional Mathematics Education in the USA

The criticism of mathematics education came from several areas - from colleges, society and education in general. Colleges complained of the inadequate knowledge of mathematics of high school students. Max Beberman listed the following inadequacies of college freshmen at the College of Engineering of the University of Illionois:

> 1) poor computational facility, 2) poor conceptual background, 3) ignorance of proof and structure except in geometry, and 4) ignorance of contemporary applications in natural science, social science, and technology. (NCTM (60) p. 69)

That means that the syllabus was out of date and not adjusted to new developments and applications of mathematics. In the same way as the syllabus was insufficient compared with the needs of the college was it also insufficient for the needs of society. Mathematics had become more important for common citizens. Besides the outmoded syllabus there was another alarming state. The number of pupils who were enrolled in certain courses in mathematics had decreased in those days.[1)]

Year	Algebra	Geometry	Trigonometry
1928	35.2	19.8	1.3
1934	30.4	17.1	1.3
1949	26.8	12.8	2.0
1952/53	24.6	11.6	1.7
1954/55	24.8	11.4	2.6

(excerpt from Brown (7) p. 366)

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1) The following table shows the "percent of pupils in the last 4 years of puplic secondary schools who are enrolled in certain courses in mathematics." (Brown (7) p. 366)

From the insufficiencies described above follows that the mathematics courses had to be revised not only in respect to the demands of colleges, society and economy but also according to their attractiveness to the pupils.

A third group of people attacked the mathematics education in general, and stated that it was not only the content but methods and grouping of pupils that needed a reform. Mathematics at that time receded into the background in favor of general social aims. One of the aims of American schools is to let every child have a good education which is appropriate for an average citizens. As a consequence, gifted children were neglected. The criticism against methods was especially directed towards the exaggerated drill-method. This method was appropriate to develop mathematical skills but inappropriate to develop mathematical understanding and insight in mathematics. Mathematics education neglected the teaching of concepts and did not pay enough attention to proper motivation. Another point was the ignorance of new areas of applications in mathematics. The reason for bad results in schools was seen in the inadequate teacher training. There was a strong need for teachers with better mathematical and educational background knowledge.

2. THE BEGINNING OF THE CURRICULUM REFORM (1951-1962)

The "New Math" reform movement started in the early fifties with the beginning of the University of Illinois Committee on School Mathematics (UICSM) which is regarded as the first great reform project by many authors on the subject. A lot of other projects and incidents followed which increased the interest in mathematics instruction, showed up its inadequacies, and brought forward the demand for a reform.

In this chapter the most important happenings of this phase shall be reported. They are in chronological order

- the University of Illinois Committee on School Mathematics (UICSM)
- the Commission on Mathematics
- the Boston College Mathematics Institute (BCMI)
- the Madison Project
- the University of Maryland Mathematics Project (UMMaP)
- the Sputnikshock
- the School Mathematics Study Group (SMSG)
- the Greater Cleveland Mathematics Project (GCMP)
- the Minnesota Mathematics and Science Teaching Project (MINNEMAST).

In two digressions the most influential findings from psychology with regard to the described projects shall be considered, namely the findings of Piaget and Bruner[1] on the one hand and those of Gagné[2] on the other hand.

2.1. University of Illinois Committee on School Mathematics

The University of Illinois Committee on School Mathematics can be regarded as the first important curriculum project. It was brought into existence by the meeting of members of

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1) see pp.21-23
2) see pp. 24-26

the Colleges of Education, Engineering, Liberal Arts and Sciences of the University of Illinois with Max Beberman as its director in December 1951. The reason for this meeting was a widespread dissatisfaction about the insufficient mathematics skills of colle freshmen. Therefore, problems concerning content and methods of mathematics education in high schools were investigated. These people tried to overcome the weaknesses of secondary school programs by constructing a new four-year high school mathematics curriculum for college-capable students. This group of students had been neglected in so far as the respective syllabus for mathematics did not meet the requirements they were to face at the colleges.

The work of UICSM was guided by following six principles:

> 1) A constistent preparation of high school mathematics can be devised. 2) Students are interested in ideas. 3) Manipulative tasks should be used mainly to allow insight into basic concepts. 4) The language should be as unambiguous as possible. 5) The organization of materials should provide for students discovery of many generalizations. 6) The student must understand his mathematics. (NCTM (60) p.72)

These principles show crucial points of content and methods which will be specified in more detail.

The content of the traditional high school mathematics programs for grades 9 to 12 were oriented at those traditional expectations which the pupils were to meet at a college.

> The curriculum developer ... has in mind the traditional expectations of what a college-bound high school graduate should know of mathematics. He is expected among other things, to solve equations (singly and in systems), to use algorithms and formulas in transforming algebraic and trigonometric expressions, to deduce theorems from postulates, to find ordered pairs which belong to relations, to graph relations, and to apply his knowledge to a host of geometric and physical problems. We think these objectives are sensible and proper, and our curriculum is planned accordingly. (NCTM (60) pp. 253-255)

Special importance was given to the understanding of mathematics. In order to meet this demand, UICSM tried to make the language used in mathematics texts more precise. Of particular note, UICSM included a resident mathematician who developed a tree-diagram system of proof which incorporated all the logical strategies needed in mathematics. As soon as the student learned about algebraic generalization, he was shown here to use the proof system to draw conclusions.

By 1958 the first courses were fully developed. Materials were arranged in eleven units. These units covered the range of content of the former secondary school programs but presented them in a new way. In order to bring about more precision, students used a deductive approach and set terminology. Other innovations included

> 1) the move toward an integrated mathematics curriculum in which algebra is found throughout the four-year program; 2) the fact that geometry, especially solid geometry, is minimized; 3) the disappearance of trigonometry as a seperate subject and the introduction of the more general circular functions; 4) the introduction of some modern concepts such as set terminology and quantifiers; and 5) the relocation of several topics, such as mathematical induction and inequalities, to lower levels in the curriculum. (NCTM (60) p.254)

Following topics were covered by the eleven units:

1. Arithmetic of the Real Numbers
2. Generalizations and Algebraic Manipulation
3. Equations and Inequations
4. Ordered Pairs and Graphs
5. Relations and Functions
6. Geometry
7. Mathematical Induction
8. Sequences
9. Elementary Functions
10. Circular Functions and Trigonometry
11. Polynomial Functions and Complex Numbers

(NCTM (60) p.253)

These topics are sequentially arranged.

Because of the need for understanding of mathematics, discovery teaching and learning was instituted as a superior

method in mathematics teaching. In order to understand mathematics, the students have to play an active role in developing mathematical ideas and methods. It is a characteristic of the UICSM program that the student first learns to understand mathematics. He does not verbalize his discoveries until he is ready. It is this point that distinguishes the discovery method as seen by the UICSM from other views of discovery methods.

In order to use these courses, specially trained teachers were required. Until 1958 the UICSM texts were only given to those teachers who had attended a special course to be properly prepared for teaching such programs. UICSM staff trained teachers in "summer institutes", where these teachers attended a summer course lasting six weeks. In these courses they learned about new content as well as new methods.

UICSM's teacher training includes: developing a strong background in mathematics; informing about principles, programs and methods of teaching of mathematics; guiding teachers in teaching UICSM materials in participating schools; demonstrating lessons in front of interested groups; and the preparing of films that show the use of UICSM materials in junior high school classes.[1)]

Besides the construction of courses and work in the area of teacher training, UICSM experimented with a course in programmed instruction during 1961 through 1964. After 1964 it was engaged in three new areas of curriculum development: 1) teaching fractions to low-achievers using the "operator method" for grades 7 and 8, 2) Geometry for junior high school students - this course was at first thought of as a course for low-achievers but eventually became a supplementary course for all students, and 3) vector geometry for highly talented students and teacher students at colleges.

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1) see NCTM (60) pp. 252 f.

Later on UICSM was occupied with computer assisted instruction.[1] The work of UICSM ended in 1972. During the last years the emphasis lay on teacher education. At the outset UICSM was financially supported by the Carnegie Corporation of New York. USOE took over this task from 1962 until 1964, and NSF sponsored the project until its end in 1972.

UICSM presented a model for following projects, e.g. SMSG. Its influence was not limited to the USA. Its work effected the reform movement in the Federal Republic of Germany and many of its results can be found in school books in West-Germany, for example, the operator methods, logic or set terminology.[2]

2.2. Commission on Mathematics

The Commission on Mathematics is not a project but a commission that was convened by the College Entrance Examination Board (CEEB) in 1955 to examine the Secondary-School-Mathematics Syllabus of that time and to recommend improvements. The main reasons for the concern were the test-scores gained by the Educational Testing Service (ETS) in 1954 and other investigations of problems within mathematics education, especially those related to college-capable students. The group consisted of equal numbers of people from college mathematicians, high school teachers, and lecturers in mathematics education, who came from all over the United States. Their task was to set up a syllabus which met the level of student achievement required by the demands of mathematics, natural sciences, social science, economy, technology and industry in the second half of this century.

The commission's main concern was program content, but compared with traditional programs there were only slight changes. The greater changes were in the abstractness of

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1) see chapter 3.5.
2) see Steiner (84) p.137

the treatment and in presenting topics earlier in the students' schooling. All recommentations were summarized in a program for college-capable students:

> Strong preparation, both in concepts and in skills for college mathematics at the level of calculus and analytic geometry.
> Understanding of the nature and role of deductive reasoning - in algebra, as well as in geometry.
> Appreciation of mathematical structure ("pattern") - for example, properties of natural, rational, real and complex numbers.
> Judicious use of unifying ideas - sets, variables, functions, and relations.
> Treatment of inequalities along with equations.
> Incorporation with plane geometry of some coordinate geometry, and essentials of solid geometry and space perception.
> Introduction in grade 11 of fundamental trigonometry - centered in coordinates, vetors, and complex numbers.
> Emphasis in grade 12 on elementary functions (polynomial, exponential, circular).
> Recommendation of additional alternative units for grade 12: either introductory probability with statistical applications or an introduction to modern algebra. (NCTM (60) pp.260 f.)

More detailed comments were established for single courses for grades nine to twelve.

Little was said about the methods of teaching by which the recommended content should be taught to students. But there were some comments in the commission's report that conceal some methodological recommendations. One example:

> Most if not all of the Commission members would prefer to see a developmental approach, which would encourage the student to discover as much of the mathematical subject matter for himself as his ability and the time available (for this is a time-consuming method) will permit. (NCTM (60) p.262)

The Commission on Mathematics contributed a great deal to the reform movement. Its final report is said to be the most carefully reasoned out and influential document of this kind. As a result of publications of these recommendations a general interest in the reform of curricula arose all over the country. Many projects were influenced by these

recommendations. Among them one should not forget to mention the School Mathematics Study Group which will be described later.

2.3. Boston College Mathematics Institute (BCMI)

Lecturers of the mathematics department of Boston College, esp. Father Stanley Bezuska, made inquiries about the knowledge of mathematics of college freshmen and discovered large deficiencies. To do something against these deficiencies the Boston College Mathematics Institute was founded in 1957. Its aim were pre-service and in-service teacher education in the field of theoretical aspects and use of mathematics of that time. At the same time new teaching materials were prepared for students from grades seven to twelve.

The project's main concern was the content aspect of mathematics education. The stress lay on a structural approach to mathematics as well on the historical aspects of this subject. Teacher education was directed at content, theory and terminology of "Modern Math". No attention was given to the methodological aspect.

The activities of BCMI include:

1) Written materials for college and secondary school level. The prime concentration consisted of texts for junior high school as well as a basic text in "Modern Math" for teachers. The junior high school texts were part of a series called "Contemporary Progress in Mathematics" and were mainly designed for use by teachers with little knowledge of "New Math". Some topics were whole numbers, rational numbers, euclidian geometry, numeration systems, and set theory. The books were written in such a way that the students could read them for themselves and work on them without any further help. There was one course for the seventh grade for low-achievers in mathematics. For teachers with little knowledge of "Modern Math" some texts were written for unaided work (Cooperative Unit

Study Program, Courses I and II). There was also a calculus text, a text on modern mathematics for parents, a text about computer oriented mathematics and several other teaching aids. 2) Institutes were provided for in-service teacher education. These institutes ran NSF-summer courses and academic year courses. Demonstration lessons were an integrated part of these courses. In addition BCMI organized workshops in different parts of the USA, including California and Hawaii. Workshops are courses for teacher education which are held in single schools or school districts in certain time intervals. 3) Computerprograms for high schools were developed. Within these programs students were taught the fundamentals of computers as one field of applied mathematics, and they were also taught how to work with computers. These courses were constructed during the second phase of the reform movement.

2.4. Madison Project of Syracuse University and Webster College (Madison Project)

The Madison Project was established at Syracuse University, New York, in 1957. Robert B. Davis was appointed as director. Another center was opened at Webster College, St. Louis. The project got its name from Madison High School of Syracuse where the materials were first introduced. The project got financial support from NSF and various other foundations such as the Helzer Foundation of New York, the Alfred P. Sloan Foundation and a group of industrialists and commercial companies around St. Louis.

The aim of the project was to foster more effective learning, to develop, to disseminate and to implement a supplementary program in mathematics for kindergarten and grades one to twelve, which would enable children to have creative learning experiences. Davis regarded the aim of building up readiness in the use of mathematics as the project's main task.

> This, then, is the first task that the Madison Project has set itself: to study high school and college mathematics, determine what concepts and what techniques are most nearly essential, and to build readiness for them over a period of years, instead over a period of days or weeks, as is normally done at present. (Davis (19) p.572)

The aims concerning the syllabus were

> 1) to broaden the curriculum to include relationship of arithmetic to algebra and coordinate geometry, 2) to instill a more creative flavor into the school mathematics curriculum, and 3) to achieve greater variety in the children's experiences with mathematics, and more active student participation. (Sherman (82) p.22)

Finally three basic working hypotheses should be mentioned:

> 1) In general, considerable experience with basic concepts and techniques should precede any formal instruction. 2) The ability to discover patterns in abstract material is one of the most essential mathematical skills. 3) Content and method are equally important and inseperable. (Sherman (82) p.22)

When one reads these aims and hypotheses, the main concern of this project becomes clearer. Unstructured tasks help the student learn to solve problems. By means of discussions and solving such problems he is able to discover mathematical structures by himself. The attempt is to provide a learning atmosphere which lets the student think that he is the first one to deal with those problems and which lets him discover mathematics in an active way. This situation is best without a teacher's interference. This discovery process is supported by related experimenting materials, stories and games. With regard to the content the mathematical structures are stressed. Arithmetic, algebra and geometry are taught at the same time. The project's efforts are directed primarily at college-capable students.

In order to bring about an appropriate spatial background, "Math Labs" were established on following grounds:

> a. Following Piaget, it is assumed that actual perception and actual active manipulation of physical materials contribute to concept formation, for some students if not for all students;

b. The evidence seems to indicate (rather strongly) student preference for this method of learning;
c. Watching a child manipulate physical materials often gives the teacher deeper insight into how the child is thinking about a task than can be obtained by purely verbal methods;
d. Traditional verbal methods seemed to create a superficial rote learning that did not seem to help the child when - as in a shop or lab - he was confronted with tasks involving real objects;
e. Using actual physical objects necessarily challenges the "one right answer" syndrome that afflicts many children and teachers;
f. "Math labs" create a desirable classroom social setting for individualizing instruction. (Davis (20) pp.41 f.)

Mathematics laboratories were equipped with computing machines, overhead-projectors, film strips, tape recorders, measuring instruments, and other teaching materials which facilitate discovery learning.

Within the scope of the work of the Madison Project one must not forget to mention those materials for experimenting which were included in the laboratory equipment. These materials were designed to promote a high level of discovery and problem solving. Games and stories were produced for use with those materials.

Two particularly significant texts were published. The first for grades 4 to 8, "Discovery in Mathematics", deals with topics in axiomatic algebra, coordinate geometry and applications in the field of natural sciences. The second text, "Exploration in Mathematics", for grades 6 to 9, treats topics in statistics, logic, matrix algebra and applications in physics.

In order to teach those materials, one needed specially trained teachers on whom one could make high demands. Because of this, workshops were organized in various large towns, and a broad advisory system was arranged. Many films were made which demonstrated the application of materials in teaching situation. Within in-service teacher education two courses were developed using films, written materials and laboratory equipment.

This approach in this project differed considerately from others. Whereas the emphasis lay on improving or introducing new content in other projects such as the UICSM and BCMI, the methodological aspect was noticably placed into the foreground within this project. Ideas by Piaget and Bruner, an advocate of discovery learning, yielded the psychological basis for the mathematics laboratories.

Digression: Psychological base (Piaget and Bruner)

Piaget worked on child development. For him the desire to learn depends on the child's maturity of body and mind. He developed a theory which defined three stages of the child's mental development which shall be shortly described by the following quotation.

> Basically, the mental development of the child appears as a succession of three great periods. Each of these extends the preceding period, reconstructs it on a new level, and later surpasses it to an ever greater degree. This is true even of the first period, for the evolution of the sensori-motor schemes extends and surpasses the evolution of the organic structures which takes place during embryogenesis. Semiotic relations, thought, and interpersonal connections internalize these schemes of action by reconstructing them on the new level of representation, and surpass them until all the concrete operations and cooperative structures have been established. Finally, after the age of eleven or twelve, nascent formal thought restructures the concrete operations by subordinating them to new structures whose development will continue throughout adolescence and all of later life (along with many other transformations as well).
> The integration of successive structures, each of which leads to the emergence of the subsequent one, makes it possible to divide the child's development into long periods or stages and subperiods or substages which can be characterized as follows: (1) Their order of succession is constant. ... (2) Each stage is characterized by an overall structure in terms of which the main behavior patterns can be explained. ... (3) These overall structures are integrative and non-interchangeable. (Piaget/Inhelder (74) pp.152 f.)

From this theory several principles can be derived for the teaching of mathematics. Progress in learning has to remain within the limits of the respective stage. New content

ought to be introduced by means of situations in which only some aspects are new to the child and which otherwise offer as many starting points as possible for the application of things already known. In this connection activities have to be organized which make the pupil tackle the problem intensively. Particularly during the first stages the teacher has to start from concrete objects and activities.[1)]

Mathematics laboratories offer the elementary school child active interaction with new objects out of which the child can develop mathematical concepts in an extraordinary way.

Next Piaget, Bruner's theories are most important for the teaching of mathematics. He also worked on the development of cognitive abilities. Intermittently, he worked together with Piaget. Bruner stressed the importance of structures in learning. He declared that knowledge is subdivided in structures. The acquisition of knowledge is the same as building new structures. A child with low cognitive structure must be assisted in such a way that the child's structures will correspond with those"scientific structures" as they are found in ideas and concepts. In order to arrive at such a correspondence, a certain way of proceeding is necessary, namely one that starts from the child's concepts and arrives at the desired correspondence by assimilating the concepts. Bruner called this process "spiral approach". "Das Spiralmodell bedeutet inhaltlich, daß die Sachverhalte periodisch auf jeweils höherem Niveau der Erkenntnis von neuem behandelt werden" (Keitel (43) p.22)

> Auf unteren Erkenntnisebenen werden mathematische Sachverhalte empirisch im Umweltbereich des Schülers entdeckt. Später wird schrittweise analytisches Denken entwickelt und die axiomatische Methode zur Darstellung und Analyse mathematischer Strukturen herangezogen. (Keitel (43) p.22)

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1) see Wittmann (92) pp.60-65

Bruner advocates "Discovery learning" as one way for acquiring such structures: "Forschend gelangt der Schüler in den Besitz von Strukturen, und als Forscher verhält er sich wie der Wissenschaftler, der die Strukturen beherrscht." (Keitel (43) p.22)

From these considerations Brunder's well-known hypothesis becomes clear:

> We begin with the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development. (Bruner (9) p.33)

This means more precisely:

(1) Der Unterricht ist in jedem Fach in erster Linie auf die fundamentalen Ideen ("Struktur") der jeweiligen Fachwissenschaft auszurichten. D.h. der Mathematikunterricht ist in erster Linie auf solche Grundideen auszurichten, welche eine ökonomische Beschreibung und Erklärung von Sachverhalten und Zusammenhängen in der Wirklichkeit und auch innerhalb der Mathematik erlauben.

(2) Die Grundideen eines Faches können jedem Kind, gleich welcher Altersstufe oder sozialen Herkunft, auf der Grundlage der Denkmittel, die es mitbringt, und der Darstellungsmittel, die es versteht, in entsprechend einfacher Form vermittelt werden. (Wittmann (92) p.67)

Bruner developed a "Medientheorie der psychologischen Repräsentationen" (Ellrott, Schindler (25) p.73),[1] where he distinguished the three modes of representation: enactive, iconic, and verbal-symbolic. Each mode of representation has its own way of describing the proceedings and of making them understandable. These modes differ from each other by the level at which things are represented.

After this detailed digression there is still one aspect of the Madison Project which should not be omitted. There are not materials except the two described above which refer to specific grade spans, but they can be used in teaching

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1) theory of modes of representation of material from a psychological point of view

as supplementary materials in all grades for stimulating and motivating.

The Madison Project was widely noticed because of its profound proposals and multi-purpose materials. It influenced such projects as the English Nuffield Project and later on, the results of the Cambridge Conference on School Mathematics, "Goals for Teachers in Elementary Schools" and "Goals for the Correlation of Elementary Science and Mathematics" which were to be published in 1967 and 1968.

2.5. University of Maryland Mathematics Project (UMMaP)

The University of Maryland Project was started in fall 1957. It was one of the few projects in this phase of the curriculum reform which was guided by a behavioristic conception, which means

> in ihm wird versucht, eine Gesamthierarchie mathematischer, behavioristischer Lernziele zu konzipieren und in Curricula der Programmierten Instruktion umzusetzen. Es wird versucht, die "Taxonomy of Educational Objectives" von B. S. Bloom in geeigneter Weise auf mathematische Ziele und Gegenstände zu beziehen und in logisch und lerntheoretisch sinnvollem Bezug als eine Abfolge von Grob- und Feinzielen zu konkretisieren.
> (Keitel (43) p.46)

The applications of psychology in this project came from Gagné's learning theory. Gagné himself was one of the members of this project.

Digression: Gagné's learning theory

Gagné's theory says that knowledge is psychologically organized in a hierarchy and that before one can learn more complex things one has to learn simple principles and concepts. His hierarchy of learning consists of eight types of learning.

> In order of increasing complexity, the eight types are: signal learning (Learning a response to a signal, as in classical conditioning);

stimulus response learning (Learning a connection between a response and a discriminated stimulus);
chains (Chains of two or more response connections);
verbal associations (Learning of chains that are verbal);
multiple discrimination (in which the individual "learns to make n-different identifying responses to as many different stimuli");
concept learning (The acquisition of "a capability of making a common response to a class of stimuli that may differ from each other widely in physical appearance");
principle learning ("A chaim of two or more concepts");
problem-solving ("A kind of learning that requires the internal events usually called thinking"). In thinking, two or more previously acquired principles must be combined to produce a new capability. (Mussen, Conger, Kagan (54) p.64)

Each type of learning is a prerequisite for a successive kind of higher order of learning.

With these types of learning Gagné combines internal and external learning conditions. Internal learning conditions depend on the learner, for example on his efficiency and already available prerequisites. On the other hand external learning conditions are called forth by the stimulating situation. According to Gagné the results in learning are dependent on previous learning. In this connection he stresses controlled teaching.

Unterrichten heißt, die Bedingungen des Lernens, die außerhalb des Lernenden bestehen, zu arrangieren. Diese Bedingungen müssen stufenweise aufgebaut werden, wobei auf jeder Stufe die eben erworbenen Leistungsmöglichkeiten des Lernenden, die Erfordernisse für die Erhaltung dieser Möglichkeiten und die spezifische, für die nächste Lernstufe nötige Reizsituation gebührend beachtet wird. (Ellrott/Schindler (25) p.92)

When building syllabi according to Gagné one starts from structures and fills them with the respective content.[1)]

For such arrangements taxonomies of educational objectives are of great help. In the first quotation of this chapter Keitel refers to Bloom's taxonomy of educational objectives.

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1) One can find an example for a structure for adding whole numbers in Ellrott, Schindler (25) p.93

Bloom and his colleagues constructed two taxonomies. In this context only the first is of interest as it deals with educational objectives in the cognitive field. This taxonomy consists of six main groups: knowledge, comprehension, application, analysis, synthesis, and evaluation, all of which are again subdivided into subgroups forming a hierarchy.[1)]

Several departments of the University of Maryland[2)] and various school districts around Washington D.C., took part in UMMaP. The project was conducted by a committee whose members came from the participating university departments together with the representatives from the Maryland State Department of Education, the Maryland State Teachers Association, the USOE, the cooperative university departments and the public schools. In the beginning John Mayor was the leader of the project. Financially it was supported by the Carnegie Foundation, USOE and NSF.

The project's aim was

> to determine maturity levels at which certain mathematical concepts can be appropriately taught, and to prepare materials for a teaching sequence in grades seven and eight which is mathematically and psychologically sound and appropriate to modern-day needs. (Keedy ([42]) p.281)

Besides this objective the project aimed - especially later - at pre-service and in-service teacher education.

The materials for grades seven and eight were written by 25 teachers during weekly seminar sessions after these teachers had taken part in lectures about mathematics and psychology. Later, these teachers tested those teaching units which were experimental in their classrooms.

In the beginning UMMaP based the content of its materials on topics from traditional mathematics. But soon these were

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1) see B.S.Bloom (6)
2) Department of Mathematics, Education, Psychology, and Engineering

replaced by new topics and with new approaches to the old ones. Not only calculation ought to be taught in mathematics education, but students should also gain a knowledge of the structure of mathematics. Concepts were stressed by encouraging thinking of and in mathematical structures.

The following materials were written:

1) "Mathematics for the Junior High School: First Book" and "Mathematics for the Junior High School: Second Book". In these books the language and the structure of mathematics is stressed. The symbols and their corresponding mathematical concepts are strictly differentiated. The importance of various numeration systems and of a consistent development of more complicated numeration systems is accented. New topics are statistics, probability theory, logic and trigonometry.
2) The book for teachers "Mathematics for Elementary Teachers: Book I" is about number systems in arithmetic. Its sequel "Mathematics for Elementary Teachers: Book II" is about geometry, especially measuring.

An "In-service Course" for Elementary School Teachers was developed in 1966 and 1968 and sponsored by USOE. In the eady seventies a new course was constructed for in-service education of Elementary School teachers which stressed mathematical processes and discovery-learning in small groups.

The program of UMMaP was used among others as information for writing a corresponding course, namely for grades seven and eight, by SMSG.[1)]

2.6. Sputnikshock

On October 4, 1957 the first artificial satellite, called 'Sputnik', was sent into orbit by the USSR. This event caused a sensation all over the world, especially in the United Staes of America. The USA had been striving

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1) see p. 32

for leadership in the world against the USSR as far as armament and scientific development was concerned. The launching of 'Sputnik' affected the USA's self-confidence deeply because it appeared to the rest of the world that the USSR was now holding the first place in the field of science - in this case in space exploration - which is closely connected with research in the military field. Because of this event faith in the American school system was shaken in the USA. Furthermore, the Americans saw their national safety threatenend. Even so, both aspects also brought the problems of American schools before the public at the same time.

> Sputnik focused public attention upon the problems of education in a highly dramatic way. (Mueller (53) p.696) The Sputnik incident fostered an acute awareness of serious danger to our national security. (Rosenberg (79) p.723)

Some reports, mainly from the USA, declared that this 'Sputnikshock', as it is sometimes called, was the releasing factor of the reform movement. In any case, as described above, the reform had already started before: UICSM had been running since 1951, since 1955 the Commission on Mathematics was dealing with problems of the teaching of mathematics, other projects as UMMaP, BCMI and Madison Project had just started their work. The 'Sputnikshock' brought the problems of mathematics education before the public. Thus it exerted pressure on the government to take part in the reform by supporting it financially. After all, the USA had to fight for its image and position in the world.

> The cry, 'We dare not fall behind the Russians', sometimes darkly hinted, sometimes blatantly affirmed, because something of a touchstone that unleashed torrents of money for the purpose of bringing about reforms in science and mathematics instruction. In all, it was a classic instance of a right response for a wrong reason. (Mueller (53) p.696)

On the part of the government increased efforts in the field of education were the result. Even in 1958 an act was enacted - the National Defense Education Act

(NDEA) - which provided financial support for a period of four years. Part of this money was used for research. It made several projects in mathematics and natural sciences possible.[1] This financial aid also made the acquisition of technical renewals since there were laboratories and other media in schools possible.[2]

There were also other ways of supporting projects from the side of the government. The NSF - following the government's example - also asked for money for the development of new programs in mathematics and natural sciences for the first time in 1958. This money was used in the following years to support most of the projects.

2.7. School Mathematics Study Group (SMSG)

The SMSG was the biggest of all the projects. Its materials were the most widely spread. Furthermore it was this project that was taken as a model for many text books in mathematics education. Its occurrence on such a large scale was probably due to the 'Sputnikshock' of the preceeding year, which had forced the Federal government in Washington to provide large sums for the reform movement. SMSG got considerable financial support by NSF until its end in 1972.

The SMSG came into being because of recommendations of two conferences which took place in the beginning of 1958. The first of them was the 'Chicago Conference on Research Potential and Training' which took place at Febr. 21, 1958. Its task was to investigate problems of supply and demand of research mathematicians. In its resolution the conference asked the president of the American Mathematical Society (AMS) after consulting the presidents of the Mathematics Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) to convene a committee of

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1) see Keitel (43) p.3
2) see Beckmann (9)

mathematicians who would try to find a solution to the problems of the school mathematics syllabus. The second conference, which was at first called 'Mathematics Meeting of the NSF' and later on the 'Cambridge Conference' took place at the Massachussetts Institute of Technology in Cambridge at February 28, 1958. Its members were mainly research-mathematicians who discussed together with physicists the Physical Science Study Committee (PSSC) - a project founded in 1956 and which aimed at improving the teaching of physics. They were informed about the recommendations of the Commission on Mathematics.[1)]

This group also formulated as a resolution the call for a setting up a new committee with following tasks:

> 1) That a four- or five-week writing session be held that summer (1958) to prepare a detailed syllabus for a model secondary school program beginning with the seventh grade and 2) that a series of monographs on mathematical topics of interest and value to secondary students be published. (NCTM (60) pp. 272 f.)

This committee was nominated by the AMS president on April 3, 1958. This event was the first time in the history of AMS that it officially expressed its interest in the mathematics syllabus of schools. The committee consisted of eight members (for this reason it was also called 'Committee of Eight'.) Its leader was E.G. Begle and its location was Yale University, New Haven, Conn. On May 5, the NSF delivered the first money where upon the committee's actual work started. The Committee of Eight appointed an advisory committee which consisted of twenty-six persons from high schools and colleges from all over the USA.

During the following summer the first texts were written by a work group. In the first year it consisted of forty-five people, among them twenty-one college teachers and supervisors as well as respectively one representative of Rand Corporation, Bell Telephone Laboratories and of the American Association for the Advancement of Science (AAAS). This heterogeneous group became typical of all following work

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1) see p. 16

groups. Because of its size this group was divided into subgroups, one for the writing of texts for grades seven and eight and respectively one for grades nine to twelve. All these subgroups formulated their texts according to a consensus which had been arrived at within each group and oriented at the principles and aims of SMSG. This large amount of freedom within the single groups caused problems of coherence among the texts. The merit of these groups was a good cooperation among the members who came from such many different fields. But it was just this cooperation which was one of the aims of SMSG.

Its main aim was the improvement of the school mathematics syllabus. The SMSG started its work by writing texts for college capable students at secondary schools. These texts were to be used by commercial text book publishers to develop classroom materials. Subsequently they were retired from school use. For this reason the SMSG's work was confined to designing materials. The SMSG itself did not make any efforts to introduce its materials into schools on a broad scale. On the contrary, SMSG restricted the scope of innovations in such a way that all its teaching materials and accompanying teacher's editions expected little background knowledge on the part of the teacher, just what could be learned in a six-week summer-course or a regular course. These courses took place once a week for one year before using these materials for the first time.

The mathematical aims of the SMSG project reflected in the content of this syllabus were based on the demands of the society. One can get an impression of them in the following quotation:

> The world of today demands more mathematical knowledge on the part of more people than the world of yesterday and the world of tomorrow will have still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; and

> understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship. Since no one can predict with certainty his future profession, much less foretell which mathematical skills will be required in the future by a given profession, it is important that mathematics be so taught that students will be able in later life to learn the new mathematical skills which the future will surely demand of many of them. To achieve this objective in the teaching of school mathematics, three things are required. First, we need an improved curriculum which will offer students not only the basic mathematical skills but also a deeper understanding of the basic concepts and structure of mathematics. Second, mathematics programs must attract and train more of the students who are capable of studying mathematics with profit. Finally, all help possible must be provided for teachers who are preparing themselves to teach these challenging and interesting courses.
> Each project undertaken by the School Mathematics Study Group is concerned with one or more of these three needs. (NCTM (60) p.275)

The SMSG worked on the basis of the recommendations of the Commission on Mathematics except for the field of geometry. Beside this basic work its members were well acquainted with UICSM materials and the speaker of the group for the Junior High School course was also director of the UMMaP. He brought in some of the items of the project which were personally guided by him.

The SMSG syllabus encompasses both the traditional and the new topics as appropriate. The traditional topics are approached in new ways, and there is simple opportunity for practice and drill of mathematical skills. The main characteristics are the stress on the structure of mathematics, the 'set concept' and the 'spiral approach'. With regard to the methodological aspect the SMSG used, in addition to the spiral approach, the deductive approach and occasionally the discovery approach. The SMSG also accents the part of intuition. The authors themselves did not give any explicit information about psychological influences.

The SMSG developed materials on a pretty large scale. Its work did not only concentrate on the writing of text books but also included materials for in-service teacher education, monographs, reports about the project and its

work, surveys and test series. The number of its publications amounts to more than three hundred.

Originally the text books were designed for college-capable students in secondary schools. Only later were lower grades and lower achievement groups included. The text books were designed according to following principles:

- Der Mengenberiff durchdringt alle Zweige der Mathematik in allen Klassen; die Begriffe und Symbole der Mengentheorie werden fortschreitend und in gemäßigter Form durchgeführt.
- Das Verstehen der mathematischen Strukturen wird stark betont. Die Grundbegriffe mit ihren Eigenschaften und wechselseitigen Beziehungen bilden logische Systeme; diese verbinden isolierte Fakten des Lehrstoffes, Themen, ja sogar ganze Abschnitte der Mathematik miteinander.
- Der Begriff der Operation wird verallgemeinert; es werden Operationen mit einem Element, mit zwei und mehr Elementen untersucht; es werden ihre Eigenschaften untersucht, Kommunitativität, Assoziativität, Distributivität, Identität, inverse Elemente, inverse Operationen. Die Operationen werden nicht auf Operationen mit Zahlen beschränkt.
- Die Schüler werden eingehend mit verschiedenen Zahlensystemen bekanntgemacht, einschließlich der Ausführung von Rechenoperationen in diesen Systemen, und zwar hauptsächlich deswegen, damit sie das dekadische System gründlich verstehen lernen. Dabei wird auch die praktische Anwendung des dyadischen Zahlensystems bei Rechenmaschinen nicht vergessen.
- Das Messen ist ein weiteres vereinheitlichendes Prinzip, das im gesamten Mathematikkurs immer wieder auftritt. Der Schüler soll die Wichtigkeit der Maßeinheiten begreifen, die Zufälligkeit ihrer Festlegung; er soll schrittweise den Begriff Größe kennenlernen, ferner den Abstand zweier Punkte, die Größe eines Winkels, die Größe weiterer geometrischer Gebilde.
- Grundsätzliche Änderungen wurden in der Algebra durchgeführt, sie wird als eine einheitliche logische Struktur mit einer relativ kleinen Anzahl von Leitgedanken dargestellt. Die Schüler lernen schrittweise die Eigenschaften der Begriffe Gruppe, Ring und Körper kennen.
  Die Vektoralgebra bildet ein selbständiges mathematisches System, sie wird zur Lösung vieler verschiedenartiger Probleme benutzt.

(Jelinek (38) p.353)

Moreover, many text books contain historical remarks wherein they illustrate the usefulness of mathematics. From these remarks they give an illustration of the use of mathematical theory within examples from natural

sciences, technical science, etc. These principles convey a good picture of main points in the scope of the syllabus. To illustrate the following brief descriptions of SMSG courses in chronological order are given.

The junior high school course for grades seven and eight is regarded as foundation for the following high school courses in algebra and geometry. In the high school courses innovations include the introduction of the set concept and terminology, emphasis on the function concept, gradual development of the properties of real numbers and emphasis on the organization and presentation of content. The main text for grade nine stresses algebra. A one-year introductory course in geometry was assigned for grade ten. For grade eleven a course in intermediate mathematics was designed which covers the same topics as usually found in a traditional syllabus (for example, trigonometry, vectors, logarithms, inducation and complex numbers) with decreased emphasis on calculation skills. Two courses were planned for grade twelve. These courses were 'Elementary Functions' for the first half of the year and for the second half an introduction to Matrix Algebra. For more talented students a further course on probability theory and statistics was designed. All these courses were ready for use in 1960.

The elementary school courses were written in 1962 as programs for grades four to six. By 1964 programs for Kindergarten to grade three were completed. These text books were also restricted to average and above average students. This elementary school program stressed mathematical structures and concepts, an early introduction of Geometry and an extension of parts of Arithmetic. Additional chapters were added for more talented students.

In 1962 SMSG published its first texts for less able students in grades seven, eight and nine. The original programs for grades seven to nine had been revised and rewritten for students with a lower level of achievement. For low achievers in Kindergarten and grade one appropriate texts were published in 1966.

By that time text books had been written for all grades and now SMSG started to revise its Secondary School Program and to adjust it to the latest mathematical needs of society. Aims of this new curriculum called 'Secondary School Mathematics' differed considerably from those of the original program, which mainly prepared students for college. This new course was devoted to students of all ability levels.

> This curriculum should be devoted almost entirely to those mathematical concepts which all citizens should know in order to function satisfactorily in our society.
> The traditional grade placement of topics should be ignored. Instead, topics from arithmetic, algebra, and geometry should be interwoven in such a way that they illuminate and support each other.
> The basic ideas about certain new topics, such as computer mathematics and statistics, should be made available to all students.
> It is important to make clear to all students that mathematics is indeed useful; that it can help in understanding the world we live in and in solving some of the problems that face us. (Greenberg (36) p. 642)

In writing these books SMSG advanced more carefully than it had previously done in developing the original books and also prolonged its experimental periods. Between 1964 and 1966 three supplementary courses were designed for the upper grades. These courses were a one-term course in analytical geometry (1964), a course on calculus (1965) and a one-term course on computer mathematics (1966).

In close collaboration with natural scientists four units were produced in 1964 and 1966 which illustrate simple scientific experiments. These units were to motivate and to lead to the formation of mathematical concepts. Three of these units present physical experiments and are intended for junior high school students. The fourth unit refers to biology.

Many of the text books were translated into several other languages - in particular, Spanish for schools in Puerto Rico. The other translations were texts for such

foreign countries as Sweden, Turkey, Taiwan, Australia, Brazil.

Single topics of mathematics are treated in the monographs. The main aims of these monographs are

> to disseminate good mathematics at the secondary school level which will supplement the usual high school curriculum; to awaken interest among gifted students; and to present mathematics as a satisfying, meaningful human activity. (Begle (3) p. 242)

They serve as supplementary materials for students, teachers and other interested readers.

Together with each text book there is also a commentary for the teacher. These commentaries include the required background knowledge in mathematics, teaching suggestions and answers to the problems. Furthermore, a series of books titled 'Studies in Mathematics' was prepared to give further background knowledge of certain SMSG texts. Another series of 'Study Guides' for various topics of mathematics was developed which was intended to help teachers to improve their knowledge of mathematics. There was one film course for elementary school teachers which was made in 1963 to give those teachers a better understanding of the underlying mathematical concepts taught in elementary school.

In 1964 SMSG tried to bring its courses into a programmed form, often spoken of as 'programmed learning'. The algebra course for grade nine was rewritten as a learning program and evaluated. The other courses which followed were calculus, algorithms; computation, a course in computer mathematics, pre-school programmes and a course 'New Mathematics for Parents'.[1)]

The SMSG conducted empirical surveys and tests to judge the effectiveness of its programs as well as to gain new impulses about the nature of new programs from the results. Other studies examined the programmed algebra course and the film course for elementary school teachers. A two-year

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1) see Keitel (43) p.43

survey investigated the achievements in mathematics of junior high school students. In 1962 one survey was aimed at problems of identifying specially talented students. In 1967 another survey was conducted which aimed at programs that were directed towards this same group of students.

Its most important survey, known as the National Longitudinal Study of Mathematical Abilities (NLSMA), was started in 1961 and became one of the major efforts in evaluating the effects of the mathematics revolution. NLSMA's main purpose was

> to investigate the relationship of textbook usage and the broad collection of mathematical ability, achievement, attitude, and psychological variables measured in the study. (CBMS (17) p.110)

Since fall 1962 NLSMA identified and tested three groups of students:

> The X-population, consiting of 38,000 fourth graders in 1962, was tested every year for five years; The Y-population, consisting of 48,000 seventh graders in 1962, was tested every year for five years; The Z-population, consisting of 24,000 tenth graders in 1962, was tested every year for three years. All subjects were from schools that volunteered for the study and SMSG had no control over the textbooks or instructional methods used in the classes. (CBMS (17) p. 109)

For the analysis grades 4, 5, and 6 were grouped, grades 7 and 8, and each of the grades 9 to 12 was analyzed seperately.

The main conclusions of the study at grades 4 to 6 are the following:

> First, after achievement measures have been adjusted for a reasonable comprehensive set of initial conditions, many significant differences between textbooks groups remain. Second, between the behavioral levels of computation and comprehension there were radically different patterns of achievement among the textbook groups...

> From a somewhat different point of view, the study showed that while there is a clear trend for modern textbooks to be associated with poorer performance on computation scales, the modern textbooks themselves are associated with widely varied patterns of performance on the other scales. (CBMS (17) pp. 110 f.)

The following general conclusions were suggested at the seventh and eighth level:

> First, students who used a conventional mathematics textbook series in the seventh and eighth grades did relatively well on test items dealing with computation and relatively poorly on test items that require the more complex abilities of comprehension, application, and analysis. Though the SMSG group achievement fitted the opposite pattern, the different modern textbooks exhibited quite dissimilar profiles of performance on many of the mathematical scales. Second, the results of both X- and Y-population analyses at grades 7 and 8 support one of the guiding hypotheses of NLSMA: that mathematics acievement is a multi-dimensional phenomenon. (CBMS (17) p. 111)

Results for grades 10 to 12 showed fewer clear trends of achievement associated with textbook classification. NLSMA give as one explanation the fact that senior high school teachers are generally far better trained in mathematics than their elementary or junior high counterparts.

The extensive data on attitudes and many psychological attributes of students revealed only few relationships between these variables and basic mathematical achievement. NLSM observered that it is very likely that the instruments for measuring affective variables may not have been appropriate or sufficiently sensitive.[1)]

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1) see CBMS (17) pp.112 f.

Even at the outset the importance of the SMSG within the USA was considered. Its materials were wide-spread at the national level and also beyond its borders. They were used in many other countries. They also influenced the teaching of mathematics in the Federal Republic of Germnany indirectly by the OECD to which the SMSG materials had served as a model for its 'Synopsis of Modern School Mathematics'.[1)]

2.8. Greater Cleveland Mathematics Project (GCMP)

The Greater Cleveland Mathematics Project, which later on became known as 'Educational Research Council Mathematics Program' (ERCMP), was set up as result of an inquiry of superintendents of twenty-one school districts at the Educational Research Council of Greater Cleveland in 1959. It got financial support by several smaller foundations among which were the Grant Foundation and the Lubrizol Foundation. This project was guided by a behavioristic conception. The aim of this project was

> das gesamte Mathematikcurriculum in einer umfassenden Abfolge aus logisch aufeinander abgestimmten Materialeinheiten für alle Klassenstufen vom Kindergarten bis zum zwölften Schuljahr zu entwickeln, das sowohl den Standards der Entwicklung der Wissenschaft in den mathematischen Stoffen und ihrem Aufbau als auch den Bedingungen einer behavioristischen Lernzielkonstruktion und Operationalisierung genügen soll. (Keitel (43) pp.46-47)

The following goals should be achieved by

> 1) the traditional program's sole objective - to develop skillful and rapid computation at the elementary school level and to develop skillful and accurate manipulation of algebraic expressions at the secondary level, and 2) to help the child to discover for himself basic principles that enable him to recognize mathematical structure and thus solve related problems. (Sherman (82) p. 21)

The GCMP's work was based among other things on the recommendations of the Commission on Mathematics.

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1) see Keitel (43) p. 44

The emphasis in the content of this program concerns the handling of logical structures and their relations. Further, the project deals with traditional as well as new topics; it is oriented towards concept formation; and it regards arithmetic as a fundamental part of mathematics. Moreover, the GCMP uses set terminology and distinguishes between symbols and concepts. The acquisition of computational skills plays an important role.

The 'discovery approach' is regarded as the appropriate way of teaching mathematics.

> The GCMP attempts to lead its students to the challenging and exciting phases of mathematics-in-the-making by having them rediscover the fundamental patterns and by having them recreate the basic symbolism, thus making the learning of mathematics to a large extent selfmotivating. (Grundlach (37) p. 192)

The methods used for teaching are manifold. They range from individual studies via work in laboratories, lectures, films, seminars, discussion groups on to the use of computers. Individualized teaching methods are used mostly.

The GCMP also laid great importance on teacher education. In the beginning an extensive, well-organized in-service education program was designed which encloses lectures, seminars, pertinent materials and series of TV education programs, demonstration lessons as well as slide-tape programs about special topics and approaches. These teacher-education courses were written in the form of programmed instruction. Besides this program the teachers were given the opportunity to attend summer courses and workshops.

In the course of the project development special branched programs for less able students and slow-learners were developed. After 1970 the development of individualized teaching programs became the central point of GCMP's efforts. This was also the cause for the exploration of

computer-assisted instruction.[1)]

This project was the first one which had planned a comprehensive mathematics curriculum for all grades.

2.9. Minnesota Mathematics and Science Teaching Project (MINNEMAST)

The Minnesota Mathematics and Science Teaching Project started in 1961. It was first headed by Paul Rosenbloom who was followed by J.H. Werntz. Originally the project started on a small scale and was funded by means of a NSF-grant for the SMSG. The following year the University of Minnesota directly received sufficient funding which allowed the project to continue and extend its work.

The project's goal was to design a coordinated mathematics-science curriculum and an experimental training program for pre-service teacher education. The curriculum was addressed to elementary school students and emphasized development of the following four goals:

> 1) to know how a scientist acquires knowledge; 2) to observe, classify, generalize, predict, and test prediction; 3) to recognize problems and discover answers; and 4) to build a coherent structure of scientific knowledge. (NCTM (58) p. 28)

In terms of content the following topics were emphasized: real numbers, euclidian space (in an abstract sense) and a measurable space (concrete applications of planes, volumes). Arithmetic and geometry were taught at the same time so that the child could learn a deductive way of proceeding.

Methodologically MINNEMAST stressed the discovery, observation and exploration of mathematical patterns. Games, stories and other devices served as motivating materials. The teacher should use the child's curiosity by asking about the child's discoveries and discussing them. Even in the books for upper grades the emphasis was

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1) see Keitel (43) p. 48

on concrete experiments and activities which provide a basis for developing mathematical insights and ideas. This methodological approach showed a strong influence of Piaget's theory of developmental stages.

In the sequel of its development, after 1963, MINNEMAST oriented its work to the Cambridge Conference on School Mathematics' recommendations. At that time MINNEMAST tried to connect mathematics and science education at elementary level in a single curriculum. Mathematics teaching was designed around the learning of natural science.

MINNEMAST produced twenty-nine sequenced units for grades K - 3 together with corresponding teacher and student textbooks. One idea which should be mentioned is that of a mathematics library. The children were given books which they took home in order to build up their own libraries. By proceeding in such a way the parents also had the chance to become acquainted with new materials.

In the areas of teacher education MINNEMAST developed a 'College-level Geometry Package' for pre-service high school teacher education which contained filmed lessons, programmed units for studying and extending texts, and problem materials. A similar course in geometry was designed for in-service teacher education.

MINNEMAST came to an end in 1970. Its materials can still be found in a few schools.

2. 10. Comprehensive Survey of the First Phase

The first efforts of reforming mathematics instruction in the schools came from the colleges as one can see by the leadership of the projects. With the years the gap between college and school mathematics had become increasingly larger, while the fields of application of mathematics increased even more. With this growth the demands on mathematics in other subjects also increased. In order to diminish this gap, the first activities came into being. In accordance with the interests of those groups it followed that the first efforts were aimed at the

mathematics syllabus for secondary schools. It was set up especially for those students who were planning to attend college. The Sputnikshock then caused a public interest in the need of a reform of the entire mathematics curriculum. At this time other interests became involved: the increasing demand for mathematically trained manpower, the growing importance of mathematics for each and every citizen in a technological society, and last but not least, the struggle between the USA and the USSR for the leadership of the world. The Federal government in Washington reacted to the public pressure by passing the National Defense Education Act (NDEA) which opened the doors to a fundamental reform by providing large sums of money. Shortly thereafter the biggest and most influential project, the School Mathematics Study Group, came into being. This project also started at the secondary school level. It was not until later projects (GCMP and MINNEMAST), that the reform turned to the elementary school curriculum. It was finally realized that the work on elementary school level had to be regarded as the basis of an effective work on secondary school level. Although the elementary school curriculum was regarded now as well, the main efforts were still aimed at average and above-average students. Before the early sixties the less-able students were not being considered.

This reform of mathematics teaching includes modifications of its goals, contents, teaching methods and teacher education, as well as the new image of mathematics within the entire school curriculum. For some time before this reform the emphasis had been on 'social development'. Objectives and subject matter held a subordinate position. This hierarchy changed because of the increasing importance of those disciplines within society and within education.

In terms of content the changes involved the arrangement and description of topics. These changes in most projects were not caused by introducing new topics but by a new arrangement of the old as well as a shift in emphasis within

the traditional syllabus. Traditonally mathematics topics were taught diconnectedly and the main emphasis was on acquisition of computational skills. Now, the aim was a coherent, logically structured syllabus in which not only computational skills could be learned but the student would also understand mathematics. In order to reach this aim, the mathematical structures and concepts were stressed. Set theory and logic were used for a more mathematically accurate presentation. The following topics were used to reach this aim:

Structure
Operations and their inverses
Measurement
Extensive use of graphical representation
Systems of numeration
Properties of numbers, development of the real number system
Statistical inference, probability
Sets - language and elementary theory
Logical deductions
Valid generalizations (NCTM (60) p. 282)

Many of these topics reflected the developments within mathematics of the recent decades when arithmetic, algebra and geometry were taught simultaneously. Now it was necessary to restructure parts of the curriculum. In contrast to the traditional curriculum where only geometry was taught deductively, proofs were now extended to all parts of the mathematics curriculum. The new topics besides set theory and logic were those which were related to applied fields of mathematics such as probability theory, statistics and computer mathematics.

On the elementary school level the structures within arithmetic became even more important. Some topics were taught which usually belonged to the syllabus of upper classes. Some of them are:

1. A substantial amount of intuitive, nonmetric geometry
2. Nondecimal systems of positional notation
3. Expanded notation and its application to algorithms
4. Exponential notation
5. Open sentences using placeholders
6. Inequalities
7. Coordinate geometry
8. Number theory, particularly primes (NCTM (60) pp.138f.)

On the secondary school level the courses in intuitive geometry and algebra were offered for grades seven and eight, and were regarded as basic to the following courses. Two other topics within these courses were simple concepts of probability theory and statistics. A formal course in elementary algebra was usually planned for grade nine, a geometry course for grade ten and for grade eleven a continuation in algebra, trigonometry and a first introduction to analytical geometry. In grade twelve followed circular functions, logarithmic and exponential functions, polynomials and sometimes probability theory and statistics. The more talented students could even work on a one-year calculus course.[1)]

The construction of the syllabus followed various principles. Those projects which were oriented at a behavioristic conception, such as UMMaP and GCMP, arranged the topics within a previously constructed hierarchy in which there was a certain place for every concept or skill. The SMSG proceeded on the basis of the spiral approach.

Whereas originally the methodological aspects were often neglected, they were considered increasingly during the course of the reform. The UICSM developed a particular form of discovery learning. The Madison Project regarded both the methodological and the content aspects as equally important. This project also emphasized discovery learning in a manner similar to GCMP and MINNEMAST. The tendency left not doubt that all efforts were being made to eliminate the deficiencies of the traditional mathematics curriculum. Now the student was to take an active part. He was not expected to listen to the teacher as much as he was expected to discover mathematics for himself. With this he ought to understand mathematics better and to be motivated as a result of his own activities. On the elementary level there were concrete experiments and activities out of which

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1) see Ferguson (32)

mathematical concepts and insights should be developed. On the secondary school level students should work increasingly on an abstract level.

The curriculum reform was influenced by the latest findings within learning and developmental psychology. During the first phase this influence was restricted to only a few projects. This situation arose from the fact that most people concentrated their efforts on the content of mathematics teaching. Even so, in those projects which they did influence, they caused a drastic change. The child's learning development was brought into the foreground after which the content was presented by appropriate methods. Some examples of this change were the mathematics laboratories of the Madison Project, the restructuring of topics according to the spiral approach of SMSG, emphasizing discovery learning in many projects and also the structuring of entire curricula according to hierarchies of learning. Hereby the scope of the reform was immensely broadened. It was not only necessary to find new topics, but more important to structure and to present them adequately. Besides the new demands from mathematics and society the child, who had often been subject to out-moded content and bad teaching, had to be taken into consideration.

The combined efforts of the mathematicians, the experts in the teaching of mathematics, the mathematics teachers and the educational psychologists were signs of a large-scale attempt to account for all aspects of mathematics teaching at schools scientifically. All the projects were initiated by colleges and universities and guided by the advice of 'scientists' of various disciplines including industry and other professional branches. The general aim was the construction of an optimal curriculum, which would measure up to the requirements of all participating sides (various subjects, education, psychology and vocational interests).

Among other things a reorientation in teacher education was combined with the reform. In order to introduce the new materials, specially trained teachers were usually

needed. There were efforts to improve pre-service teacher education. The credential requirements especially in the training of elementary school teachers were found to be absolutely insufficient. In mathematics the prospective teacher usually had to attend just one arithmetic course. The demands on future high school teachers were higher but because of the teacher shortage at that time, many less-qualified teachers were employed. It was difficult to agree on standards because pre-service teacher education differed regionally. More important for the introduction of new materials was the in-service teacher education. This was done by means of workshops, summer courses and academic-year courses. Beginning in 1953 the NSF had been organizing such courses. As already mentioned, some of the projects themselves took over this task. They produced materials for in-service teacher education, organized courses and demonstration lessons and also arranged information centers.

Most of the projects did not cross regional borders. There were just a few that were carried out on a national level, for example the Commission on Mathematics and the SMSG.[1] Unfortunately there were almost no studies at all which provided detailed information on the distribution of materials. The USOE published a survey which dealt with trends in mathematics course offerings and enrollment over a period from 1948/49 to 1962/63. About seven per cent of all schools were covered by this survey. From a table which compared the 'Number of Pupils and Percent of Increase in Certain Public High School Mathematics Courses'[2] with the total number of pupils at the age of fourteen to seventeen, one can deduce a growing interest in mathematics from the increased number of enrollments. In the same article Brown and Abell refer to 'New Math' programs. About twelve per cent of those schools offered such courses for grade ten, about seventeen per cent offered courses

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1) see pp.37 f.
2) Brown, Abell (8) p.654

in "Integrated Geometry" and less than six per cent of the schools offered reorganized courses for grade twelve. For schools with more than 1500 students, the following numbers show slightly better results: about a third of them offered new courses for grades nine and ten and about one fifth of them offered such courses for grades eleven and twelve.[1] This means that the availability of new courses was relatively low in this phase. In this connection one should not forget the difference of involvement in the reform movement between large and small schools.

After the first materials of SMSG, UMMaP and GCMP had been introduced into the schools, there were a series of smaller surveys which tried to compare traditional and new mathematics curricula by comparing annual achievements in mathematics. In general no significant difference was to be found when stardardized tests had been used. Even so, it must be admitted that achievement levels of pupils who were taught mathematics according to traditional textbooks in arithmetic were better than that of pupils who had been taught mathematics according to new programs. On the other hand the better pupils demonstrated better achievement in modern terminology and concept formation.[2]

These efforts of reforming the school mathematics syllabus were severely critisized. One of the first, most persistent and most fierce opponents was Morris Kline of Courant Institute of Mathematical Sciences, New York University. He did not deny the need for a reform, but directed his criticism at the way at which it proceeded. Already in 1958 he published an article in which he critisized the recommendations of the Commission on Mathematics.[3] Even more attention was given to a second article which he published in October of 1961. In this article he refered to the SMSG program. His criticism was directed to the following concerns:

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1) see Brown, Abell (8)
2) see Williams, Shuff (90); compare pp.37 f.
3) see Kline (44)

> It contended[1] that much of algebra, geometry, and trigonometry were replaced by set theory, symbolic logic, matrices, and Boolean algebra; that all the material was arranged in a formal structure; that recommendations were made casually; that the work was done helter-skelter; and that the reformers asserted that they had the correct curriculum. In the judgement of the writer of that article, the problem of mathematics education was not that the curriculum was outmoded, but rather that the presentation of materials was very bad - that there was little motivation, little intuitive development before generalization, no inclusion of applications, and little active participation on the part of the students in creating the material he was to learn. (NCTM-(60) p. 82)

According to his view a reform had to start with the teachers who could produce more efficient learning by better ways of teaching. In his article of 1958 he mentioned four principles which, as he saw it, could be regarded the basis of a new curriculum.

> My first principle, then, is that we must begin our teaching of mathematics by seeking to arouse interest in the subject...My second principle is that regardless of how we arouse interest in mathematics, even if it be through games, we must supply motivation and purpose to the mathematics we teach... Third, since even the great mathematicians think intuitively, we must all be sure that the intuitive meaning of each mathematical idea or procedure is made intuitively clear to the students... Fourth, knowledge is a whole, and mathematics is a part of that whole. Mathematics in every age has been part of the broad cultural movement of the age. We must relate the mathematics to history, science, philosophy, social science, art, music, literature, logic, as well as to any other development which the topic in hand permits, We should try as far as possible to organize our material so that the development of the mathematics proper is related to the development of our civilization and culture. (Kline (44) pp. 425 - 427)

In 1962 a memorandum was published which had been signed by sixty mathematicians of the USA and Canada, among them Morris Kline. This memorandum refered to the SMSG's program

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1) 'It' refers to Kline's article

and to the recommendations of NCTM from 1959. Its criticism included the following items: Similar demands in mathematics for all pupils, although these demands were oriented at future mathematicians and scientists; warning against early formalism and introduction of too difficult mathematical concepts; criticism of a deductive approach, formal proofs and too rigid use of the distinction between symbol and concept at the cost of application and educational factors.[1]

Finally, a thorough commentary on possible disadvantages of this 'new mathematics' as L.B.Kinney saw them is given.

- Disregard of the Purposes of Secondary Education. No consideration is given to future needs of the pupil, or what contributions society will require from him... Data on what can be taught need to be supplemented by information on what should be taught, in the light of the purposes of the secondary schools, before the program of curriculum development can be completed.
- Neglect of Important Concomitant Outcomes. One of these is development of skill in the techniques of problem solving in a real rather than an abstract problem situation... Emphasis on the structure of mathematics, however, has led to de-emphasis on application... Thus, while discovery and generalizations in the area of abstract mathematics are encouraged, the development of systematically applied techniques of problem solving is neglected.
- Neglect of Differential Needs of Various Pupil Groups. The purposes for teaching mathematics must take into account the abilities, interests and vocational needs of various groups of pupils, not only as a service to the pupils, but also for the benefit of society, which needs the fully developed resources for each group. (NCTM (60) p. 286)

It is the expressed opinion from these facts that these points are partly justified. Originally the reform concentrated too much on talented pupils and in this connection stressed formal, abstract approach too much. This way of proceeding can be justified only for those students who will continue in mathematics at the college level. One must not forget that only a small number have this aim. Although

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1) see 'On the Mathematics Curriculum of the High School' (70)

the reform later on turned towards less able students and slow learners, the programs for these groups were still derived from those for talented students. By over-emphasizing the theoretical aspect of mathematics, applications in every-day situations were neglected.

On the other hand, every criticism must be looked at carefully. It can happen that different projects are 'teared all with the same brush'.[1] There are some elements common to every project, but also every project has its peculiarities and its own approach by which it contributes to the reform. And also, in rewriting and in creating new activities each project demonstrated its commitment to learn from the past.

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1) see as an example Kline ([44])

3. THE ZENITH OF THE CURRICULUM REFORM (1963 - 1971)

The beginning of the second phase can be fixed at the time the Cambridge Conference took place, although it is impossible to draw an exact line between the phases. At that time new emphases and endeavors arose. The reform became more comprehensive, the people were enthusiastic and optimistic. There were many groups now willing to contribute financial support to the reform movement.

After the curricula of the first phase, during which projects mainly concentrated on the innovation of content for average and above-average students, other kinds of curricula began to appear. Those may be divided into three categories: projects which make use of computer-assisted instruction, projects for less able students and inter-disciplinary projects.

This chapter includes a description of the following events:

- Cambridge Conference on School Mathematics (CCSM)
- Secondary School Mathematics Curriculum Improvement Study (SSMCIS)
- Comprehensive School Mathematics Program (CSMP)
- Contribution of the Federal Government: R & D Centers ESEA
- Computer-based Mathematics Instruction at the Stanford-based Laboratory for Learning and Teaching
- Project LOCAL
- Project PLAN

  } Computer-Assisted Instruction

- Central Iowa Low Achiever Mathematics Project (CILAMP)
- Committee on Low Achievers in Mathematics - Denver Area (COLAMDA)
- Jefferson County Public School Mathematics Program

  } Projects for Low Achievers

- Portland Project
- Interrelated Mathematics - Science Project
- Unified Science and Mathematics for Elementary Schools (USMES)

} Interdisciplinary Projects

3.1. Cambridge Conference on School Mathematics (CCSM)

The Cambridge Conference on School Mathematics arose from an initiative at a meeting of members of the Massachussetts Institute of Technology, Cambridge, together with the NSF in 1962. Those participants had accepted the invitation of two professors, J.R.Zacharias and W.T.Martin of M.I.T., to discuss the state of mathematical instruction in the schools of that time. All participants agreed that far more radical revisions of the mathematics curriculum were necessary for the following years. In order to discuss such future reforms and their problems, the CCSM was organized. This conference took place in Cambridge, Mass. from June 18 until July 7, 1963. Twenty-five mathematicians as well as representatives of universities and industry who were interested in the application of mathematics in their respective fields such as statisticians, physicists, chemists and economists, participated in this conference. It was headed by the professors W.T.Martin from M.I.T. and A.T.Gleason of Harvard University. Among the participants were E.G.Begle, director of SMSG, R.B.Davis, director of Madison Projet, P. Suppes, director of the Computer-Based Mathematics Instruction Project, and Jerome Bruner. It is remarkable that nearly all members came from universities or were of a similar status. From this situation an effective as possible discussion should be guaranteed.

Within this conference the goals were to be discussed for the teaching of mathematics for the years 1990 to 2000.

> What was now required were the outlines of a new reform program which may begin to become effective when current tasks have been accomplished and a new generation of mathematics teachers has begun to appear. Consequently,

> it was decided that the conference should deal primarily with the goals of school mathematics, leaving aside the relationship of these goals to existing educational resources. (Goals for School Mathematics (35) p.2)

The participants agreed that the problems of that time should be neglected on purpose.

> They ignored the problem of teacher training, they ignored the fine points of pedagogical technique, and they did not take account of recent research in cognitive learning. They agreed to be bound only by the limits of their own imagination. (NCTM (60) p.291)

The results of that conference were summarized in a well-known and widespread report, "Goals for School Mathematics". The claims of this report and the hopes combined with it were explicitely laid down in the beginning.

> At the outset, it is important to recognize that this report is a discussion document, and not a prescription. It would certainly not be appropriate, here and now, as a guide for general teaching, if only because one would be able to find so few teachers capable of teaching it. At this stage of the effort at least, the aims are more modest: it is hoped that our intentions and initiatives will command this report to all who think about mathematical education, whether or not they are actually engaged in experimental work in this field. (Goals for School Mathematics (35) p.1)

The statements made in the report can be roughly divided into the following three categories:

1) Statements about fundamental goals of modern school mathematics;
2) Statements about pedagogical principles and techniques;
3) The setting up of a curriculum for grades K to 12.

CCSM proposed the following frame of content for the twelve grades including Kindergarten:

> The subject matter which we are proposing can be roughly described by saying that a student who has worked through the full thirteen years of mathematics in grades K to 12 should have a level of training comparable to three

> years of top-level college training today; that is, we shall expect him to have the equivalent of two years of calculus, and one semester each of modern algebra and probability theory. (Goals for School Mathematics (35) p.7)

In order to master all these topics within the thirteen years which traditionally ranged over a period of sixteen years the following proposals were made:

> It is clear that the inclusion of more content at the top must be compensated by the omission of something else. We propose to gain three years through a new organization of the subject matter and virtually total abandonment of drill for drill's sake, replacing the unmotivated drill of classical arithmetic by problems which illustrate new mathematical concepts. (Goals for School Mathematics (35) p.7)

After setting up those topics, the general objectives for such a comprehensive curriculum were specified. As it was mentioned above, drills, which only served the purpose of practising skills, were rejected. In their place it was suggested that adequate practice of arithmetic computation should be woven into the acquisition of the concepts, which is with regard to meaningful material.

Further the CCSM recommended working for the aim of making the student familiar with mathematics in the course of mathematical instruction straight from the beginning. In order to reach this aim, the spiral approach was recommended where geometry and arithmetic (later on algebra) were taught simultaneously.

Mathematics was also seen as part of a liberal education. One important aspect within the teaching of mathematics was to build up self-confidence on part of the student. The CCSM believed that in classes where the discovery approach was used even the less able students could achieve good results in mathematics if they were familiar with just a few basic ideas. But for this approach it was important for the student to have sufficient self-confidence.

The concepts from modern mathematics were recommended because of their usefulness for the organization of the

topics. The employment of mathematical terminology and the mode of expression was to serve a more precise formulation and treatment of mathematics. The CCSM refered in this connection to an accurate examination of each single term and concept, because an introduction of too many minor ones would cause needless embarresment. Side by side with theoretical mathematics the applications of mathematics were not forgotten. Because of this situation the CCSM supported the idea of introducing topics by means of their applications.

During the course of the mathematics lessons the students should acquire an understanding of the power as well as the limitations of mathematics. In order to reach these goals as mentioned above, the CCSM attached primary importance to the selection of appropriate material and methods of presentation.

> In reaching the goals discussed above, the selection of germane material and the method of presentation are of prime importance. An ommission of a subject from a student's curriculum can be made up readily later, in college or adult education courses, if the student has previously developed a sound approach to mathematics. On the other hand, improperly taught material may confuse the student's understanding of the facts, inhibit good mathematical reasoning, and lead to dislike of the whole subject. (Goals for School MAthematics, (35) p.13)

One of the most important characteristics of the CCSM proposal was the introduction of the spiral approach.

> The use of a spiral curriculum, in which the same subject arises at different times with increasing degrees of complexity and rigor, offers many advantages. At the first stage an intuitive or pre-mathematics approach offers the opportunity of an early introduction of important concepts. There is time for each of these concepts, first drawn from the student's general experience, to be made more familiar and more precise, and time to develop the concept further. The concept can be used by the student from the beginning in appropriate simple contexts.
> ...After an interval in which the concepts have become more concrete in the student's mind through use in other contexts, the subject is brought back to the student's attention with a greater degree of formal organization. (Goals for School Mathematics (35) pp. 13 - 14)

In order to make the student think independently and creatively, the following methods were proposed:

1) The discovery method , in which the student is asked to explore a situation on his own. This approach results in a large sacrifice of time.
2) The aiding of discovery where the teacher provides the pupil with considerable variation in the amount of feedback and reinforcement to guide the student.
3) The group discussions where each student can make a contribution to the discussion and thereby learns to defend his point of view as well as to be able to revise it appropriately. The teacher, for his part, must control the better students if necessary or let them speak at the end in order to avoid the situation where less able students will be intimidated by them.
4) The exercises make up the most important part of a course. They should guide the student to the meaning and relevance of theorems and concepts, to train and to develop skills, creative thinking, and ingenuity. They can be represented in form of problems.

In order to provide motivation, the CCSM proposed the use of historical background topics to indicate the interest which was shown for various parts of mathematics and how they yielded examples of its applications. With reference to the future this approach should show the student the usefulness of the things which he was learning at the school. For elementary school pupils the mathematics laboratory was regarded as a means to stimulate interest. At secondary school level the student's interests in mathematics should be enriched by the reading of pamphlets which described special problems in mathematics.

Special achievement tests were developed to measure understanding and creativity of the students. The CCSM rejected thosetests which only aimed at measuring mechanical skills and rote responses.

Although the members of the CCSM were fully aware of the fact that especially younger pupils understand more than they can verbalize, they recommended the introduction and early practice of an accurate mode of expression. This should help the child express his discoveries and experiences in communication with other pupils and with the teacher.

The CCSM distinguished two kinds of application: The internal, which refers only to mathematics itself, and the external applications (for example, in the field of natural science). Both kinds should be used to such a degree as they helped clarifying mathematical concepts. With regard to applications in the business and consumer aspects of life, the students should learn to apply mathematics itself properly and should know what mathematicians do.

The CCSM stressed intellectual competence of students and assumed that the efficiency of teaching was mainly contingent on teachers' correct presentation of mathematics.

As addition to general aims and educational considerations, CCSM gave specifications and descriptions of the content of a mathematics curriculum for grades K to twelve. In the last part of the CCSM report there appear some interesting statements about the content of this curriculum. There is one proposal for a curriculum for grades K to six because all the participants agreed with this statement. However, this agreement did not extend over the entire curriculum. There are two versions of a curriculum for grades seven to twelve.

The objective for mathematical instruction in elementary schools is familiarity with the real number system and the main ideas of geometry. The topics were roughly divided into two parts: one for K to grade two and one for the grades three to six. In grade K to two the pupils should get their first experiences in mathematics. The content of these grades is therefore called 'premathematics'. Premathematics includes following fields:

1) An intuitive understanding of the real numbers properties is to be developed.

2) Geometry is to be studied together with arithmetic and algebra from kindergarten on. Some of the aims of this study are to develop the planar and spatial intuition of the pupil,

to provide a source of visualization for arithmetic and algebra, and to serve as a model for that branch of natural science which investigates physical space by mathematical methods.[1)]

3) Logic is introduced as a pre-requisite for mathematical ways of thinking and methods; set theory unifies all the different mathematical concepts; and functions are introduced as an important basic concept which runs through the entire mathematics curriculum.

4) Applications serve as a concrete starting point for learning mathematics intuitively.

The topics for grades three to six build on these four topics, and other pre-mathematical topics are to be initiated in these grades as a base for high school mathematics. The same topics as in the grades K to two are taught but in a more extended form; e.g. the treatment of real numbers is extended by the treatment of real number theory.

As for the two proposals for secondary school level, the first is:

Grades 7 and 8	: Algebra and probability theory
Grade 9	: Geometry
Grade 10	: Geometry, topology and algebra
Grades 11 and 12	: Analysis

The second proposal is:

Grades 7 and 8	: Algebra, geometry and probability theory
Grade 9	: Algebra, geometry and introduction to calculus
Grade 10	: Analysis, probability theory and algebra
Grades 11 and 12	: Analysis[2)]

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1) compare 'Goals for School Mathematics' (35) p.33
2) for further information see 'Goals for School Mathematics' (35) pp. 43 - 46

The first conference in 1963 was followed by two others in 1966 and 1967, where the results were published under the titles 'Goals for Mathematics Education of Elementary School Teachers' and 'Goals for the Correlation of Elementary Science and Mathematics'.

The CCSM evoked lively reactions among experts, which ranged from enthusiasm to intense criticism. A sort of moderate agreement can be found in an article by J.Fred Weaver. He writes:

> We still have much to learn about mathematics programs and mathematics instruction in the elementary school. The unanswered questions are legion. In this connection the Cambridge Conference recommendations can serve us well as a basis for formulating numerous relevant hypotheses to be tested through suitable research. From this point of view the significance of the Cambridge Conference lies in the questions it raises - some deep and profound ones which should challenge us to new and profitable research ventures.(Weaver (89) pp. 209 - 210)

One of the strongest critics was Marshall Stone. In his review of the 'Goals for School Mathematics' he did not see much good in the recommendations of CCSM.

> I have just declared my opinion that the Cambridge Report is extremely disappointing. One's first impression that the Report is daring, comprehensive, and progressive in outlook does not stand up under examination. Instead, one sees that it is really superficial, confused, and shot through with wishful thinking of a shallow kind. It is superficial in its treatment of the content and organization of the mathematics curriculum; confused in the presence of the very deep reasons for a new, modern approach to algebra and geometry at the school level; and willful in its refusal to face up to the pedagogical difficulties involved in the sweeping changes it proposes. (Stone (86) p. 354)

Although Stone was disappointed, CCSM's proposals had already been considered in other countries and were already accepted as the official curriculum in Denmark. Stone criticized the relatively small number of participating secondary and elementary school teachers, and he took a critical look at the entire curriculum. Further detail of

this criticism is beyond the scope of this work. Stone credited the CCSM

> in its willingness to challenge the extent of our currentachievements in the field of mathematical education and to demand a thorough and uncompromising revision of the entire school mathematics curriculum from grade K through grade 12. (Stone (86) p. 353)

The CCSM was also the subject of many other articles. Allensdoerfer particularly critisized the emphasis on the 'Super-elite' students and the neglect of the less-able seven-eighths of the students. Worthy of mentioning is an article by I. Adler, 'The Cambridge Report: Blueprint or Fantasy?'. He answers his own question thusly:

> The report is not and is not intended to be a blueprint for the curriculum of the future. It is a general statement of a goal to be reached, with a description of two different paths that may be followed to reach that goal, and an invitation to the teaching community to propose other possible paths. (Adler (1) p. 212)

Adler is optimistic with regard to the teachability of this program and in his opinion this curriculum can be regarded as a realistic proposal. He argued as followed:

1. The children can learn more than we think they can.
2. The transition from one stage of learning to the next can be accelerated by a better curriculum and better teaching.
3. The early use of the concepts of mathematical structure accelerates learning by simplifying and unifying the subject matter.
4. Changes like those proposed by the report have already been tried successfully. (Adler (1) pp. 214 - 215)

At the same time he describes two hindrances; namely the insufficient knowledge of many teachers and false conclusions from educational theories.

The goals set up by the CCSM were taken as a basis for various projects such as the Secondary School Mathematics Curriculum Improvement Study (1965)[1)] and the Comprehensive

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1) see chapter 3.2.

School Mathematics Program (1967)[1]. In response to the recommendations of the final meeting of CCSM in 1967, the Unified Science and Mathematics for Elementary Schools Project (1970) was created.[2] These three projects can be regarded as the most important ones of the second phase of the reform.

3.2. Secondary School Mathematics Curriculum Improvement Study (SSMCIS)

The Secondary School Mathematics Curriculum Improvement Study started its work in 1965 at the Teachers College of Columbia University, New York. The group consisted of eight American mathematicians, four mathematics teachers and four European mathematicians who had played a great role in developing mathematics curricula in their countries. Howard F. Fehr was appointed director. He himself had participated at the OECD conferences in the late fifties and early sixties, so he was well informed about European mathematics education as was another of the SSMCIS's advisors, Marshall Stone. Financial support was given by USOE and NSF.

SSMCIS's main goal was to develop and to evaluate a mathematics program for the most talented fifteen to twenty per cent of students from grades seven to twelve. Some of the program's topics are those usually taught at college. Besides this goal the others are described in the following quotation:

> It is proposed not only to create a pratical curriculum, but also to determine the teacher education in mathematics and in pedagogy necessary for the successful teaching of the program, and to develop tests and testing techniques to determine the extent and depth of mastery of the materials that are taught. (Fehr (31) p. 463)

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1) see chapter 3.3.
2) see chapter 3.7.

This project tries to break through the old system which reflects the traditional organization of mathematics into independent fields, namly one year of algebra, one year of geometry, and again one year of more advanced algebra. These fields ought to be correlated with each other. The development of courses was directed by following aims:

> The mathematics we teach toour students today should be relevant to their needs in the society of tommorrow in which they will live. ... To this end we must first of all be concerned with the development of the intellect - the ability to do cognitive thinking ... All our students must come to know mathematics as it is conceived today, what material it deals with, what types of thinking (not only axiomatic) it uses, what it accomplishes, and how it is invading almost all other domains of human activity. ... Our instruction must also have an 'informational and skill' dimension, inasmuch as it charged with transmitting from one generation to the next that inherited knowledge, and the skill to use it, that will be considered basic in the years ahead. This information should be acquired during the process of developing mathematical thinking. This target of instruction permits us to drop a great deal of the traditional content no longer considered useful and to choose more general and more unifying concepts as a foundation for instruction. ... Our instruction serves to develop the capacity of the human mind for the observation, selection, generalization, abstraction and construction of models for use in solving problems in the other disciplines. (Fehr (30) pp. 26 f.)

The SSMCIS designed six courses together with corresponding teacher's editions and five pamphlets about special fields.[1] The following list of the topics of the first course will give the reader an idea of the way as those former independent fields of mathematics are now interwoven.

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1) A summary about the topics of those single courses can be found in Fehr (30) p. 31

Organization of SSMCIS Unified Mathematics Course 1:
1. Finite Number Systems
2. Sets and Operations
3. Mathematical Mappings
4. Integers and Addition
5. Probability and Statistics
6. Multiplication of Integers
7. Lattice Points in a Place
8. Sets and Relations
9. Transformations of the Plane
10. Segments, Angles, Isometries
11. Elementary Number Theory
12. The Rational Numbers
13. Some Applications of the Rational Numbers
14. Algorithms and Their Graphs.
(Fehr (30) p.31)

In order to use these materials effectively teachers were prepared in comprehensive summer-courses which were held each year. Then these teachers introduced the courses and discussed the results of their efforts with members of the SSMCIS who then revised the courses based on the teachers' experiences.

The surveys of SSMCIS investigated the effectiveness of the courses, students' attitudes, College Board Examinations and interviews of former students who had been taught according to SSMCIS material and who had gone to college afterwards. When compared with other pupils on College Board Examinations, students who had been taught much SSMCIS materials were in no way inferior with regard to their performance. The later freshmen confirmed that the SSMCIS courses had prepared them for college very well. Contrary to traditional courses, these new methods supplied a better overall view of mathematics, making college studies easier.

The SSMCIS was a response to the Cambridge Conference on School Mathematics. It was strongly oriented to the European reform movements. This project had in Fehr's and Stone's opinion, already formulated and introduced into the schools the recommendations in the Cambridge Report.[1)]

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1) see Steiner (84) p. 138

3.3. Comprehensive School Mathematics Program (CSMP)

The Comprehensive School Mathematics Program is a continuation of the Nova Comprehensive Mathematics Project which was based on the recommendation of CCSM by Burt Kaufman at Nova High School at Fort Lauderdale, Florida, in 1963. In 1966 this project moved to and was supported and supervised by the Southern Illionois University. Thus it got the name 'Southern Illinois University - Comprehensive School Mathematics Project' (SIU-CSMP). After September 1967 the CSMP became one of the main projects of the Central Midwestern Regional Educational Laboratory (CEMREL) in St. Louis.[1)]

Originally this project was financially supported only by the participating institutions and schools. Afterwards it received support from the National Institute of Education which had been founded in 1973 by the Federal Government.[2)]

The reason for funding this project arose from the desire to improve certain aspects of the educational methods about which there was great dissatisfaction. This dissatisfaction stemmed from a number of sources: the common view of mathematics as a static subject, discontent about the conventional learning environment, available teaching materials, and insufficiently trained teachers. Thus the main aim of CSMP was

> to develop for all public and private school students, ages five to eighteen, a mathematics curriculum that provides them with a program that is sound in content, enjoyable, and appropriate to the needs and abilities of each individual child. The overriding aim of the proposed curriculum is to produce an American public that is literate in mathematics. (Exum (26) p. 538)

From the outset, the project tried to make use of the latest findings within mathematics and mathematics teaching

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1) CEMREL is one of the national 'Educational Laboratories' Which were founded by ESEA, Title VI in 1965 - see 3.4 p. 70

2) see chapter 4.1.

on the international level. In addition to the recommendations of CCSM, OECD-publications[1] influenced the work of the CSMP, as did the results of Mme. Papy's investigations in primary schools at the Centre Belge de Pédagogic de la Mathématique in Brussels.

CSMP set up general goals for the teaching of mathematics as an orientation for its work.

> We want the student to understand that mathematics is a discipline with a long history, that it is fast growing and changing, and has been for a long time too extensive for any one person to master in detail. We want him to know some of its history as it concerns major contributors and the development of important mathematical concepts.
> We want the student to enjoy mathematics, to find it worthy to study and effort, to find motivation within, as well as outside the subject.
> We want the student to understand that mathematics is useful, in direct pragmatic application, as a tool in other disciplines, and that it is a rich source of models and methods that may be used in a wide variety of ways. We want him to have first-hand experience with such uses, and be capable of using it himself.
> We want the student to learn how to obtain access to mathematics, how to find what he needs, how to go about using what he finds.
> We want the student to see, by example and direct experience that mathematics is a creative enterprise involving a free-wheeling kind of thinking and the utmost of imagination, but that the good intuitive leap or flash of insight comes to the discovering person, namely to the one who has developed a solid background. We want him to discover that his intuition can improve with use.
> We want the student to be equipped with the usual mathematical skills and competence for everyday tasks and, in particular, the skills and techniques he needs for the courses he is pursuing concurrently.
> We want the students to develop assurance in the soundness of their work, to develop resources to check their own work independently of outside authority, whether in problem solving or in writing proofs. (CSMP (15) pp.8 f.

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1) OECD. "Synopsis for Modern Secondary School Mathematics" (1961)
OECD. "New Thinking in School Mathematics" (1961)
OECD. "Mathematics Today" (1963)

The materials were designed to provide total individualization whereby every pupil could determine each step of his mathematics course himself. Despite all the emphasis on educational concerns related to mathematics teaching, the main stress was placed on the mathematics content of the curriculum.

The CSMP's work can be divided into two components

> (1) the identification and "packaging" of possible appropriate content in a K - 12 program (for children of all ability levels) and (2) the development of instructional materials for a special 7 - 12 program (for students with high verbal ability). (Status report (83) p.392)

In relation to the first component CSMP started an "Ability Package Program" for grade three in 1968. The "Ability Packages" are self-instructing multi-media teaching learning units. They

> made use of work sheets, concrete materials, games, audio and video tapes, and incorporated many new ideas on teaching arithmetic, informal geometry and probability. To each package were assigned prerequisites and goals, which were measured by pre-tests and post-tests. (Steiner (84) p.138)

The "Ability Package Program" is based on the following ideas:

> The deficiencies of teachers should be compensated by a maximal use of self-instructing multi-media teaching learning units, called activitiy packages. A curriculum matrix with content, level of abstraction, and student characteristics as its three dimensions should be designed and filled with such packages. A cybernetic system should be developed which would direct a student through a matrix along a path optimal for him, thus providing him with his own individual curriculum. (Steiner (84) p.138)

The program for the learning levels K - 2 were based on Papy's ideas. The central ideas of her program are on

> the use of various representations of numbers of sets, Cuisenaire rods, on the Minicomputer and as operators,

> the use of the Papy Minicomputer for the teaching of meaningful positional representation of numbers and calculation with rather big numbers very early, the use of the language of set diagrams and arrow diagrams to represent mathematical situations and to solve problems, early introduction of decimal numbers and their geometrical representation on the number line, beginnings of affine geometry, introduction into probability and statistics. (Steiner (84) p.139)

This program has been extended to secondary school level in the early seventies.

The second component of the CSMP's work concentrates on highly gifted students (the best ten to fifteen per cent). For them special materials were designed called "Elements of Mathematics". They assume the content of conventional high school curricula and deal with such topics which usually belong in the range of college mathematics. This series consists of following volumes:

0 Intuitive Background
1 Introductory Logic
2 Logic and Sets
3 Introduction to Field Theory
4 Order in Fields
5 An Introduction to Mathematization: A Theory of Voting Bodies
6 Relations
7 Functions
8 The Natural Numbers, Mathematical Induction and Recursive Definition
9 Finite Probability Spaces
10 Affine Geometry
11 Groups and Rings
12 Linear Algebra with Trigonometry
13 Introduction to Real Analysis I
14 Introduction to Real Analysis II
15 Field Extensions and the Complex Numbers
(CSMP (15) pp.74 f.)

These materials were tested in the USA and other countries, as well. For teachers who wanted to use CSMP materials an in-service teacher-training course was offered for the first time in the summer of 1973.

The CSMP is a program which mainly concentrates on the content of a mathematics curriculum. By means of new content and new instructional materials the sources of dissatisfaction already mentioned[1)] should be diminshed. Through these changes the CSMP tried to make it possible for the child to work under more comfortable conditions as well as to promote mathematics teaching which should be largely independent of the extent of the teacher's knowledge of mathematics and mathematics teaching.

In order to be able to carry out a program such as the CSMP, there needed to be basic changes within the school system. Some of the peculiarities of Nova School are these: the enrollment in this school is voluntary, there are no school busses, no lunch rooms, no grouping into classes, and no set schedule. The students work together in solving problems when they have reached a common level of proficiency and interest. Team-teaching is used quite often. For example, one teacher explains the problems solved at home at an overhead-projector while another one supervises the pupils singly. The children may work in small groups which are in care of other teachers who only explain those homework problems which are requested. The work at school is not tied to predetermined time periods. There is only a one month vacation per year. School facilities consist of small discussion places or seminar rooms together with learning centers which are provided with audio-visual equipment. Many of the rooms have movable partition walls, and all the rooms are supplied with overhead-projectors. Many activities occur out-doors because of the favorable climate. Each teacher has his own office with a secretary. These conditions as well as well-trained and strongly motivated teachers made an excellent working atmosphere possible.[2)]

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1) see p.65
2) see Davis (19) pp.42-46

3.4. Contribution of the Federal Government to the Reform

The Federal Government in Washington provided financial support, particularly at the beginning of the second phase when several acts were passed which made available more money for educational programs. Criticism of uncoordinated investigations and out of date research led to funding of the "Cooperative Research Program" which was established by the "Cooperative Research Act". In 1963 a national network of twenty "Research and Development (R & D) Centers" was established. These centers first undertook the coordination and standardization of individual investigations, then shortly they turned towards specialized basic research.[1)]

In 1965 the "Elementary and Secondary Education Act" (ESEA) passed and became the most important legislation after the NDEA of 1958. This act consisted of five parts.
"Title I: Grants to Local Public School Districts to Broaden and Strengthen Elementary and Secondary School Programs. This money provided for the education of disadvantaged children.
"Title II: School Library Resources, Textbooks and Other Instructional Materials."
"Title III: Supplementary Educational Centers and Services." This provided money for new teaching methods and special projects such as TV-instruction.
"Title IV: Educational Research and Training." This money supported research institutions and publication of research results.
"Title V: Grants to Strengthen State Departments of Education." This money was intended for the development of the leadership of the state departments of education within the single states. Further, it supported those programs which had the goals of identifying the needs within education and of overcoming its inadequacies.[2)]

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1) see Keitel (43) p.4
2) see Scandura (80) pp.496-498

Most of the money was administered by the USOE. ESEA made it possible by 1968 to establish about twenty regional Educational Laboratories, whose purpose was the evalutation and application of the results of R & D Centers at the regional level. Because no results from the R & D Centers existed at that time the laboratories themselves initiated basic research.[1)]

As a consequence of this federal support many new projects came into being.

> Aus all diesen Maßnahmen und Gründungen resultiert ein wahrer Boom in der Curriculumentwicklung. Entstanden 1951 bis 1963 insgesamt dreizehn Mathematikprojekte, so kommen 1964 bis 1966 fünfzehn neue Mathematikprojekte hinzu, und allein das Jahr 1967 verzeichnet zwölf Neugründungen. Allerdings handelt es sich dabei meist um kleine, kurzfristige Projekte mit festumrissenen Zielsetzungen. (Keitel (43) p.5)

Information about all these developments has been collected at the Educational Resources Information Centers (ERIC) and the Educational Service Centers which had been founded for this very purpose.

In the late sixties, at the beginning of Nixon's term of office, the first signs of a decrease of enthusiasm for the reform began to appear. The need for research in the field of education was increasingly reevaluated with regard to its urgency. These were the harbingers for the crisis in the reform which will be later explained in chapter 4.1.

3.5. Computer-Assisted Instruction

The C.A.I. projects are based on a behavioristic conception. The aim is optimal learning through individualization which uses teaching-machines, programmed materials, and computers. Together with the development of computers, advanced research was done in the field of their applications to teaching. The learning programs of the early sixties were constantly being

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1) see Keitel (43) pp.4 f.

improved technically. The computers took over certain functions of the teacher, especially the presenting of knowledge.[1)]

A great number of such projects were started and partly supervised by large companies such as IBM, Xerox, and Westinghouse Electronic Company. Some of the best-known projects are described here.

(1) Computer-based Mathematics Instruction at the Stanford-based Laboratory for Learning and Teaching

This project began in 1963 at the Institute for Mathematical Studies in Social Science at Stanford University, California, under the direction of P. Suppes and was funded by USOE and NSF. Its aim was to create computer-assisted mathematics instruction for grades one to six, and, later on, for grades seven to nine for all ability groups. As the name 'computer-assisted instruction' implies, this project uses computers for teaching mathematics with minimal knowledge of computers. The computer takes over the function of a 'guide'. An interaction between student and medium may be established in several different ways; for example, doing exercises, reading instructions on solving problems. The computer is programmed to meet the individual needs of each learner. This is possible because the computer can store and evaluate answers, providing data about the child as well as about the content of the program. Thus it can revise, extend and alter the program according to the learner's needs.

The Stanford program employs computers at three stages of teaching - for drill and practice, for presentation of knowledge and for dialogue.

The "Drill-and-Practice Program' is supplementary to the teaching of basic skills. The computer presents some

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1) for a more detailed description see under the respective projects

exercises and the learner gives the solutions. Then the computer evaluates them and presents the learner new exercises according to his performance. All the responses can be collected and reanalyzed and used as a starting point for teaching the whole class.

The 'Tutorial System' is a set of teaching units dealing with certain topics. This system uses programmed instruction on a computer which allows more interesting and exciting instructional aids than the usual programmed instruction.

The 'Dialogue System' requires a language recognition program on the computer. Verbal interaction between the student and the medium is the strategy of this program.

The materials developed in this project include the following courses and programs:

1) course in elementary logic for grades three to six,
2) course in modern algebra,
3) drill-and-practice program in elementary school mathematics for grades one to six,
4) supplementary courses in arithmetic for high school students.

The emphasis of this project is on the methodological aspect. Although this way of teaching mathematics may be quite expensive, it is certainly an effective new way to present mathematics instruction.

(2) Project LOCAL (Laboratory Program for Computer-Assisted Instruction)

This project was founded in 1967 by the city of Westwood, Massachussetts, and directed by Robert H. Haven. It was sponsored by USOE together with some local support. Here, as well, the opportunity of employing computers in teaching was stressed. The main aim of the project was to improve mathematics and science instruction through the use of computers. The project LOCAL concentrated on mathematics for grades seven to twelve. It was designed

for average and above-average students.

(3) Project PLAN (Program for Learning in Accordance with Needs)

This project, which was founded in 1967 as well, was sponsored by the Westinghouse Learning Corporation (a subsidiary of the Westinghouse Electronic Company), the American Institutefor Research and by thirteen participating school districts. It was based in Palo Alto, California.

PLAN can be described in the following way:

> PLAN is a program of individualized education that seeks to individualize what a student studies as well as the method by which he studies it. (Matthews (51) p. 689)

The aim of this project is to develop, implement and evaluate a curriculum for grades one to twelve for various subjects, including mathematics. Special emphasis was given to including as many aspects of individualization as possible.

> First, the project is using a variety of sources to attempt to define more clearly the mathematical needs of students. Second, the program develops individualized programs of studies that vary not only the content studied but also the amount of content studied for each student. Furthermore, for a given content the student is provided alternate methods for learning this content. (Matthews (51) p. 685)

PLAN starts from the assumption that not all mathematics students must have command of the same abilities and skills in mathematics. For grades nine to twelve Plan offers the following individualized teaching units which usually last about two weeks: twenty-eight in General Mathematics, thirteen in Algebra I, seventeen in Algebra II, fifteen in geometry, four in coordinate geometry, nine in trigonometry, ten in analytical geometry and fourteen in calculus.

In the development of those mathematics courses, the following sources and committees were called upon: the results from Project TALENT which is run by the same concern, a committee consisting of well-known lecturers in the

teaching of mathematics, teachers and curriculum directors of the participating school districts. From information about the mathematics units as well as the available knowledge about each single student, a certain mathematics program is proposed to each pupil. This offer not only refers to the content of the program but also to its size, the work which the student can perform in a certain amount of time. The amount of work is determined by the student's performance together with the record of his results from former work.

In order to obtain a general view of the effectiveness of a curriculum project, it is necessary to set up detailed learning goals which make an exact evaluation possible. This demand derives from a behavioristic concept, the aim of which is an objective examination of all behavior. It was above all Mager and Flanagan who worked on these kinds of problems.

With regard to the third aspect of the PLAN approach, students are offered three alternate methods to reach the instructional goals of the various teaching units. The methods differ with regard to working instructions and ways of proceeding of the pupils,[1] but the goals are the same.

With this group of curricula projects one should not forget to mention the computer-assisted instruction (CAI) developments in projects which had started during the first phase of the reform, such as BCMI, UMMaP, and GCMP.[2] The descriptions of the three projects given above give a glimpse of this kind of curricula. The part played by the big computer companies in the field of education was only a brief one. In any case these demands remained and received new impetus in the seventies in the 'accountability-movement'.[3]

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1) for examples see Matthews (51) pp. 688 - 689
2) for further information look under the respective projects
3) see chapter 4.2.

3.6. Projects for Low-achievers

As already mentioned, the emphasis during the first phase was on programs for average and above-average talented students. In the early sixties the reform turned also towards low-achievers. In the beginning of these efforts the reformers were of the opinion that those less talented students were able to learn the same things but in a longer period of time. This opinion proved to be wrong. Different content and methods had to be developed especially for this group of pupils. This task was quite a difficult thing to do because of the many different causes for low achievement in mathematics. Far less than all the students who belonged to this group had a low I. Q. Many of them simply were not interested in mathematics. They had in some cases built up an aversion to mathematics because of poor teaching. The students were often discouraged or suffered from poor reputations. Students who did not plan to go to college were enrolled in the simpler elementary courses, and thus regarded as less able. In order to do something against this abuse, brought about by the opinion that all talented students wanted to go to college and all less talented students would enter an occupation immediately after school, curriculum projects were proposed that tried to increase the prestige of courses which concentrated on certain careers and the application of mathematics to the problems likely to occur in the respective occupations.

Applications of mathematics to everyday life had been left behind in the reform because of the stress on abstract mathematics. Abstract mathematics might well be useful for able students but not for unmotivated and less able students. For these students concrete situations are required, more frequent references to them and the mathematics must show a bearing on daily life. It also is necessary to demonstrate the meaning as well as the use of mathematics for these pupils. On the other hand, everyday applications are a good

starting point to offer abundent opportunities for exploring and discovering mathematical concepts. The use of concrete materials and supplementary aids are essential to the programs for those pupils. The discovery approach is combined with the use of concrete situations. The idea of mathematics laboratories grew out of this concern for the learning needs of those students.

Ruth I. Hoffman summarized the common elements of programs for low-achievers in the following way:

> 1. A mathematics laboratory - whether it be a center for the school, a formal laboratory for the use of a few selected classes, or a classroom laboratory.
> 2. The use of calculators to help the student find his pattern of error in computation and to enable him to get past simple computational blocks to basic mathematical understandings.
> 3. A regulated program, with a pattern of activities for security but with a change of activities to accomodate the short attention-span of the slow learners and the unit-a-day pattern for the satisfaction of a task completed and evaluated on the spot.
> 4. Provision for reinforcement of early basic concepts, which may be weak, utilizing the method and techniques of the more modern programs in mathematics - the exploration of the structure of the number system, the experimentation and discovery of patterns and their utilization.
> 5. The use of many manipulative devices, such as the abacus, cuisenaire rods, geoboards, etc.
> 6. The proper and controlled use of games, puzzles, and other motivational techniques.
> 7. Use, where possible, of remote terminals tied into computers for computer-aided instruction units. (NCTM (58) p. 96)

The psychological theories of Bruner and Piaget played a role in the development of such programs. Even near the end of the first phase, several projects had turned toward the less able students. Although they developed special programs for those students, it must be admitted that the first of such programs only concentrated on the content side.

Several courseswere developed. The UICSM developed an introductory program in fractions based on the notions of operators for grade seven and an intuitive geometry course

for grade eight. In 1962 the SMSG started to design texts for grades seven to nine and, later on, for kindergarten and grade one. The GCMP also developed special learning programs for less able and slow learners. Nevertheless, before the second phase, none of the projects were especially directing their methodological point of view toward this group of students.

(1) Central Iowa Low Achiever Mathematics Project (CILAMP)

This project was conducted by Jack R. Williams at eight Des Moines Junior High Schools in Iowa. Its work started in 1967 with funding from the USOE. The project concentrated on under-achiever and low-achiever mathematics students of grades seven to nine. Its aim was to increase the interest in mathematical activities of those pupils as well as to build up their confidence in applying mathematical skills for problem solving. Teachers were required to use methods which admitted a variety of activities, provided a comfortable learning environment and used appropriate problems for the students. The whole project was concentrated on developing one laboratory classroom in each school.

On the first pages of the materials published by the project the authors describe their view of a mathematics laboratory and its objectives.

> Primarily, a mathematics laboratory is a state of mind. It is characterized by a questioning atmosphere and a continuous involvement with problem solving situations. Emphasis is placed upon discovery resulting from student experimentation. The teacher acts as a catalyst in the activity between students and knowledge.
> Secondarily,a mathematics laboratory is a physical plant equipped with such material objects as calculators, overhead and opaque projector,filmstrips, movies, tape-recorder, measuring devices, geoboards, solids, graph board, tachistoscope, construction devices, ect. Since a student learns by doing, the lab is designed to give him the objects with which he can do and learn.
> The primary goal of the lab approach is to change the student's attitude towards mathematics. Most students have become so embittered by habitual failure that they hate mathematics and everything connected with it. There

> is little possibility of this student learning mathematics until an attitude change has been effected. It is because of this goal that our approach is different. Some would label our approach as "fun and games", but I am sure that close examination will bring realization that everything in the program is oriented toward the twin goals of attitude change and mathematical improvement. (NCTM (58) p. 88)

The ideas described above were based on Piaget's philosophy of 'active learning' which maintained the student should start from concrete things and should explore his environment actively in order to come to understand it.[1)]

(2) Committee on Low-Achievers in Mathematics - Denver Area (COLAMDA)

This project had its center in Denver, Colorado, where it was implemented in near-by schools. The director was Terry Shoemaker. This project did not have its origin at a university as was the case with most other projects, but it was started by the Colorado Department of Education.

Its main task was the development of a curriculum for grades seven to twelve which concentrated on improving the mathematics performances of students without affecting their performances in the other subjects. A secondary goal was the in-service teacher education, which strove for a positive attitude on side of the teacher towards mathematics instruction.[2)]

Materials were developed for students and for teachers, but those did not take over the role of the textbook. For the students there were 'Activity Sheets', while the teachers were provided with 'Idea Cards'. All the materials were constructed to meet performance objectives and were ordered within a matrix with two dimensions: the kind of presentation, and the field of interest of the activities. The kinds of presentations included computer materials, manipulative and concrete

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1) see also pp. 21 f.
2) see Elbers (24) p. 157

materials, simulative activities, mathematical games and teacher oriented presentations. The dimension of field of interest included consumer materials, sports and recreational activities, hobbies, means of transport, and several occupations.[1)]

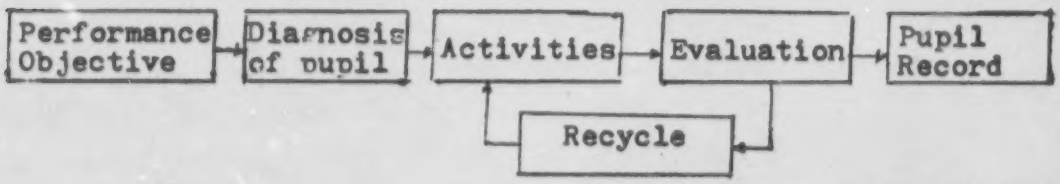
(3) Jefferson County Public School Mathematics Program

This project started at the same time as the other two described above. The Jefferson County Public Schools in Colorado worked together with the University of Wyoming Science and Mathematics Teaching Center in all phases of planning, development and implementation of a mathematics curriculum for students from kindergarten through high school.

The target students can be characterized by the following:

A record of failure in mathematics.
Achievement scores three or four years below grade level in mathematics.
Reading and comprehension difficulties in many cases.
Quick conclusions formed without due consideration of facts.
Interest span very short - 10 to 15 minutes at any activity.
High rate of absence.
Goals based on view of immediate future.
Antisocial behavior exibites in classroom and school.
Inability to see practical use of mathematics.
(NCTM (58) p. 88)

The proceedure of the project can be made clear by the following diagram:



(Kansky (39) p. 635)

The mathematics program was built on performance objectives, derived from results of research in the field of early

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1) see Sharpe (81) pp. 634 f.

childhood education and from mathematics teaching. The methods were oriented at meeting students' needs. The five main ways of proceeding could be distinguished in the following categories: (1) Multiple activities were offered. Each activity was self-contained and could be managed during a one day lesson. In order to prevent early indifference at least two changes were offered to the pupil in the course of a lesson. These activities included, among others, applications based on local business problems, special practice and multiple exercises including cross-word puzzles. The materials used were so prepared that the students' performance could be evaluated at once after use. (2) Electric calculators with tapes were provided for two students, which helped them estimate the results, discover basic relations between operations, and improve their accuracy. (3) Local business problems were presented by representatives of firms in order to motivate the students as well as demonstrate to them a field of application of mathematics. (4) The "Flow-Charting" was used to show how mathematical processes worked enabling the student to analyse and synthesize the basic processes. They started with nonmathematical concrete situations; for example, how to start a car. (5) Experiments and other activities were designed for use in a mathematics laboratory.

3.7. Interdisciplinary Curricula

The third category of new projects which appeared in the second phase consisted of those which integrated mathematics instruction with science or other subjects. The theory underlying this approach was that teaching ought to consider the personal needs of the learner as well as the subject matter objectives. Teaching should involve the real needs of present and future life together with career situations. Because such needs create problems which cannot always be solved within a single subject, cooperation is required

between the various disciplines. As the occasion demanded, the boundary lines between the respective subjects had to be removed. One form of these curricula was project teaching in which reference to real life became the most important aspect.

The most important project in this category was MINNEMAST which had already started at the end of the first phase to combine mathematics and science. Later the SMSG developed four units which considered science in the teaching of mathematics.

In the second phase there were several attempts to realize this kind of curricula. Three of those projects will be described in the following.

(1) Portland Project

This project ran from 1967 to 1971. It had been initiated by Michael Fiasca of Portland State University in Oregon. It was funded by the federal government together with some participating schools.

The project's main objective was to develop three-year programs in biology, chemistry, physics, humanities and mathematics. Those subjects were integrated in a threefold way: by using a logical sequence in content, by the use of heuristics in sciences as mediator and by concentrating on problems of the environment. In this way the students would get a more comprehensive insight about their environment than by mere repetition of learned concepts. The courses were constructed and taught according to the appropriate stages of development of the students.[1)]

(2) Interrelated Mathematics Science Project (IMS)

This project was sponsored by the USOE and the Nova Schools in Fort Lauderdale, Florida, where it was started in 1968 and headed by John E. Arena. It was restricted to grades nine to twelve. The project developed an individualized teaching program in which natural sciences and mathematics were combined as often as possible. The materials developed

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1) see Bruner's hypothesis, p. 23

were "Activity Packages" which could be used in lessons without any additional materials. The program was structured around selected problem areas in a wide variety of subjects; for example genetics, simple plants, evolution and environment, and properties of mathematical operations.[1)]

(3) Unified Science and Mathematics for Elementary Schools (USMES)

This important project started in 1970 as a result of some recommendations of the Cambridge Conference from 1967 called "The Report of the Cambridge Conference on the Correlation of Science and Mathematics in the Schools". The director was Earle L. Lomon of the Education Development Center in Newton, Mass. It was sponsored by the NSF.

This project was designed to help children of six to fourteen develop the ability to arrive at decisions about general circumstances concerning their life and society. In order to reach this aim, students were offered solvable problems of an interdisciplinary nature. Their solution contributed to the development of concepts and processes of the natural and social sciences as well as mathematics and, furthermore, to the application of heuristical methods in decision-making situations.[2)]

The concrete working objectives of USMES were following:

1. die Entwicklung von 32 Materialeinheiten, mit Lehrerhandbuch, Schülerarbeitskarten und einer Aufstellung von möglichen Lehr- und Lernhilfen oder Hilfsmaterialien, Informationen über die wissenschaftlichen Grundlagen der Problemstellungen aus den integrierten Wissenschaften (Background Papers) und Vorschlägen zur Organisation der Arbeit mit den USMES-Einheiten im Klassenzimmer.
2. die Entwicklung eines 'Super-Handapparates', der in kurzen Übersichten einen Querschnitt über Literatur zu und über USMES-Materialien liefert und durch eigene Beschreibungen ähnlicher Materialien von anderen Projekten oder Verlagen ergänzt wird.

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1) see Elbers (24) p.167
2) see CBMS (17) p.27

> 3. die Entwicklung eines 'Design-Lab', das heißt eines Laborraumes mit verschiedenem Werkzeug, Instrumenten, Geräten und Materialien, die Schüler benutzen können, um etwas zu entwerfen, zu konstruieren, zu bauen, zu messen - als Teil ihrer USMES-Lern- und Forschungsarbeit - bis hin zu Computern mit programmierten Lerneinheiten zum kurzfristigen Erwerb technischer 'skills', die während eines Problemlösungsprozesses notwendig werden könnten. (Keitel (43) p.59)

The risk of all these interdisciplinary programs is in the possibility that, after all, mathematics itself would become neglected or replaced by the other subjects.[1)]

3.8. Comprehensive Survey of the Second Phase

The Cambridge Conference on School Mathematics of 1963 can be regarded as the first project of the new phase of the curriculum reform. Although it is impossible to draw an exact line between the phases, at about this time new endeavors and emphases arose. They were based on efforts of the first phase tended to be but more comprehensive and radical. In the first phase the reform started at high school level mainly with college-capable students. In the second phase, reform started at the elementary school level and built on this basis for further programs. No longer was the main objective to overcome the gap between school and college mathematics, but the emphasis was shifted to providing appropriate mathematics education for all students according to their abilities and goals. Programs for average and above average students had to be developed as well as for less able and culturally handicapped students. The content and the methods depended on the special needs of the various groups of students.

The development and the implementation of those new curriculum directions were made possible by the "Cooperative Research Act" (1963) and the "Elementary and Secondary Education Act" (1965). In this phase, again, most of the

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1) see CBMS (17) p.63

projects were initiated in colleges and universities using the new findings of the disciplines of mathematics, learning psychology and mathematics teaching. At this time the influence of psychology greatly increased. Also, the latest developments and findings from other countries were brought to bear on mathematics programs in the USA.[1)]

With regard to content, new projects were designed to provide a solid basis of mathematical insights and knowledge at the elementary school level. This basis served as the starting point for comprehensive mathematics programs at the high school level. College-capable students were offered such courses as probability theory, analysis and even modern algebra, usually taught at the college level.

At the elementary school level basic concepts were introduced to stimulate the child's development as soon as possible. The starting point was the use of concrete situations, which gave the child a first intuitive insight into mathematics. The elementary school programs also included such topics as topology, geometry and a systematic approach to real numbers, which formerly only belonged to programs at the high school level and above.

At both the elementary and the high school level applications of mathematics were increasingly stressed. These had been neglected during the first phase because of the overemphasis on abstract mathematics. Curriculum developers came to realize the importance of references to real-life situations for students, both for better understanding of mathematical concepts and for stronger motivation to learn. The applications originated from everyday life, from natural sciences and from other disciplines which called for mathematical analyses and models. The increasing importance of applications led to closer relationships among the various subjects in the form of interdisciplinary curricula.

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1) see CSMP pp.65 ff.

Since applications contributed to a better understanding of mathematics, the programs for all the respective groups of students ought to benefit from this new approach. The reformers realized that programs at the abstract levels as found in the first phase could not be comprehended by slow and less able learners. Therefore, the special curricula designed for them stressed elementary mathematics, emphasized calculating skills and were more oriented to the mathematics needed by the average citizen. In this category were courses such as "Consumer Mathematics", which includes calculation of percentenges and interest, applied mathematics and "General Mathematics" for high school students. General mathematics deals with elements of mathematics which are not essential to further mathematical development. In these groups of courses also belong those which prepare students for the very special demands of specific careers.

Apart from that group of courses, the main emphasis was still to present school mathematics to all students homogeneously by means of introducing the basic concepts and structures, set terminology and functions. Accuracy of thinking in a logical approach was stressed.

New issues arose on the methodological side. Contrary to the first phase when the main emphasis was on stressing new content or on restructuring the old one, now the interest was focused on ways and means of teaching the given content most effectively. Hence, a greater role was played by learning psychology and technology. Increasing attention was given to providing low achievers with better methodological approaches rather than by a restructuring of content. Based on Piaget's phase theory students were to acquire their first intuitive experiences through concrete situations.

Increasingly, more importance was attached to the use of manipulative materials in mathematics instruction. Many other materials were available for enrichment of lessons, such as films, demonstration materials, supplementary books and

laboratory equipment. Discovery learning was regarded as the preferred method to help students develop their creative and natural abilities in mathematics. The teacher provided suitable aids to guide the student towards the right discoveries. But this often took up too much valuable time. So, multiple learning situations were offered to the learner: expository teaching for the entire class, working in small groups, demonstrating and discussing in mathematics laboratories and receiving individual instruction. The last one was stressed especially. It was regarded as the optimal learning method through which the student could learn according to his own abilities regardless of those of the others. Different approaches were used to provide individual learning situations. The CSMP designed multi-media learning-teaching units which could be adapted to the student's level of performance. New developments in the field of computers and their use in teaching made it possible to offer individual programs to each child electronically. Computers created a program for every single learner by considering individual and professional data together with evaluations of the student's performances.

All these endeavors of using manipulative materials or starting from concrete situations or providing individualized instruction made teaching of mathematics effective for low achievers. However, not only those students profited from these efforts to develop abilities as early as possible, but also the capable students' abilities could be accelerated.

In striving for individualization of teaching, big companies such as IBM and Westinghouse also contributed to the reform by developing soft ware for more complicated computers. Even in the long run, though, such items were too expensive. Furthermore, there was no empirically tested teaching theory in this field, and teaching through computers totally neglected the social development of the child.

Most federally funded projects operated under the condition that all curriculum materials developed were public domain and publishers were encouraged to put them to print. Royalties went to the government. Because of competition among publishers trying to keep pace with the new developments which started in the early sixties, empirical studies and longterm tests of the programs were neglected. Programs were written according to latest findings, tested, evaluated and disseminated. However, uniform comparative testing was completely missing and this was one of the big deficits of the reform. Further, empirical data about the use and the effectiveness of new programs is rare, and only vague predictions can be made about the success of the projects in new settings.

In New York and California test results were obtained for an extended period of time, and can be used to provide at least some statements of general trends resulting from the curriculum reform. In New York the performance level of the poorest twenty-three per cent of students in mathematics and reading was determined in 1966.On a comparative test in 1973 the students of grades three did better than those of the test in 1966, whereas students of grades six and nine showed poorer performances. The results for reading were no better.

In California the test results between 1969 and 1973 also showed a decrease in performance: it was quite large in the first two years but levelled out between 1971 and 1973. Upper classes showed a more moderate decrease. In California, as well, worse performances in other subjects were registered, and one may conclude that this decrease in performance was not entirely due to the new mathematics programs.[1)]

Further data about mathematics performances of pupils on a national level may be found through a study of the National Assessment of Educational Progress in the scholastic year of 1972/73. The aim of this survey was to get information

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1) see CBMS (17) pp.103-105

about the extent of understanding of certain mathematical concepts and the extent of mastery of certain mathematical skills for nine-, thirteen- and seventeen-year-old students. The study tested the students' performance in the following topics:

a. Number and numeration concepts
b. Properties of numbers and operations
c. Arithmetic computation
d. Sets
e. Estimation and measurement
f. Exponents and algorithms
g. Algebraic expressions
h. Equations and inequalities
i. Functions
j. Probability and statistics
k. Geometry
l. Trigonometry
m. Mathematical proof
n. Logic
o. Miscellaneous topics
p. Business and consumer mathematics
(Carpenter (12) p.455)

Also the following five levels of understanding and skills were tested:

I. Recall and/or recognition of definitions, facts and symbols
II. Performing mathematical manipulations
III. Understanding mathematical concepts and processes
IV. Solving mathematical problems - social, technical and academic
V. Using mathematics and mathematical reasoning to analyze problem situations, define problems, formulate hypotheses, make decisions, and verify results.
(Carpenter (12) p.455)

The results can be summarized as follows:

> The data show a mixed picture of strength and weaknesses for our mathematics programs. Students' performance was strong or at the level of reasonable expectation in terms of the mathematics curriculum for whole-number computation, knowledge of numeration concepts, analysis of simple (one-step) word problems, intuitive or practical measurement concepts, and recognition of basic geometric figures and relationships.

> Weaknesses in the mathematics program of the elementary school were indicated in the areas of percent, development of fraction concepts, complex word problems, measurement tasks, and understanding of geometry topics. (Carpenter (11) p.449)

At the high school level, students showed difficulties using problem solving strategies.

These results show a decrease in performance in certain fields of content at certain ages. This decrease of performance must be seen in context with other subject areas for which similar decreases were found. This suggests that "new math" was probable not responsible for decreased mathematics performance, and, considering all the other negative factors at work in the schools, may have been a safety net without which mathematics performance might have plummeted even lower.[1)]

Corresponding changes were taking place in teacher education. Pre-service teacher education offered the prospective teachers a better background of mathematics knowledge. Contrary to former professional training which required less than three hours of mathematics per term, these increased to three or four on the average, and even up to six. But, the emphasis was on teaching a background of mathematics knowledge, not on the theories of learning and techniques of teaching mathematics. In-service teacher education programs continued the efforts introduced in the first phase such as summer institutes, academic year courses and workshops. One successfull state-level program was the "Specialized Teacher Project", one of the four Mathematics Improvement Programs established by the 1967 session of the California State Legislature. Workshops were presented during the summer for elementary school teachers. The program was designed to

> acquaint teachers with the techniques of creating in their classrooms a climate in which pupils might more effectively learn mathematics. Instruction reflected the thinking of Piaget, professor of child psychology at the University of Geneva, and other theorists who describe a basic hierarchy in learning, from manipulation, to representation, to

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1) Another survey undertaken in the early sixties was NLSMA by SMSG; see pp.37 f.

> abstraction. It is believed that children learn mathematics, first by responding to and manipulating objects in the environment and then by reacting to representations of these physical situations before operating with the abstractions that emerge from these earlier experiences. (Dilworth, Warren (22) p.5)

The effectiveness of the project was measured in terms of pupil performance. The evaluation of the 1968-69, 1969-70, and 1970-71 workshops showed "statistically significant growth on measures of mathematical achievement by pupils whose teachers had attended the workshops and were employing an "inquiry-activity" approach in their classrooms."[1)]

In general the willingness to the reform of the second phase was pretty good. The shortcomings of the beginning tried to be avoided in new projects by revising materials. Isolated criticism contributed to these efforts as, for example, the article by H. Freemont from 1967 where he reproached the new mathematics of being too abstract and supposed the consideration of concrete situations as starting point which was done in later projects. He also did not see an end of the reform in 1967, only a step out of the first phase where it had to be attempted to overcome the first shortcomings and to strive at a better curriculum.

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1) Dilworth, Warren (22) p.5

4. DECLINE OF THE NEW MATH (1970s)

During the third phase, here called the decline of the "New Math", the reform movement in mathematics no longer got its main influence from the universities. Also the big projects disappeared. New and various societal demands and technological developments characterize this phase. This chapter deals with the significant events and movements that affected mathematics education and not with single projects nor conferences. The following major issues will be described:

- Criticism of the curriculum reform and its consequences. Mainly criticism put forward by Morris Kline and cuts in various federal grants shall be mentioned.
- The accountability movement
- Minimum competencies. Here especially the Conference on Basic Mathematical Skills and Learning and the position paper of the NCSM shall be described.
- Impact of calculators and computers on mathematics instruction
- Accountability-conscious, delivery-system oriented programs:
  - Conceptually Oriented Mathematics Program (COMP)
  - Systematic Teaching and Measuring Mathematics (STAMM)
  - Diagnostic Prescriptive Arithmetic (D.P.A.)
  - Computer Assisted Diagnostic Prescriptive Program (CADPP)

4.1. Criticism of the Curriculum Reform and Its Consequences

A strong attack on the reform movement in mathematics instruction and in other subject areas was launched during the early seventies in the form of criticism mainly on the part of the public. They echoed those criticisms spoken out by the scientists who, however, had been largely ignored up to that time. The willingness to reform diminished gradually. The expected results had failed to appear. Test scores in all subjects had become worse. The "New Math was accused of producing a generation of pupils unable to calculate. Thus, according to traditional view the "new mathematics" had failed.

Many parents despaired of the "new mathematics" which appeared to be no more than unfamiliar topics such as set theory, number systems of various bases, and abstract properties of numbers. Neither did they understand the new way topics were approached. This lack of understanding was shared by many teachers as well because they had insufficient training.

In 1973 Morris Kline, the indefatigable opponent of the reform published a book "Why Johnny Can't Add?" with the subtitle "The Failure of the New Math". This book was met with general approval. It contained a massive criticism against mathematics programs of that time, and it was directed against the whole reform. For Kline one reason for the failure of the reform was the fact that it mainly came from college and university scientists who were familiar with their subject but not with teaching. In Kline's opinion mathematicians regard mathematics as an independent system detached from the physical world. He further stated that for scientists and technicians mathematics represents an essential tool. Even so, not all pupils want to become mathematicians, scientists or technicians. Thus questions such as these arose: what knowledge of mathematics does a child really need for his/her future career? Is it really necessary for all students to be able to explain and prove mathematical procedures or would it not be sufficient for most of the pupils to be able simply to know and to apply these particular procedures and algorithms?

Kline's criticism mainly consisted of the following four points:

- novel content such as set theory, Boolean algebra, topology, symbolic logic, and abstract algebra, which is inappropriate for school curricula;
- rigorous deductive logical presentation of ideas, contrary to all historical experience of mathematical discovery;
- abstract ideas not properly grounded in concrete experience of the physical origins for mathematics;
- sterile excesses in terminology and symbolism.

(CBMS (17) p.14)

In response to the first point one should note that for

most of the population the "new math" consisted of a renewal of contents. The content introduced was then absolutely new for most people. Because of their novelty, especially, these new topics were put into the foreground of interest. They were often incorrectly treated as isolated topics which had no relationship to any others. Thus, the purpose of the new content, which actually was intended to provide uniformity of content by unifying the terminology and the use of superior structures containing the basic laws, was not realized. In the beginning of the reform people accepted any innovation designed to correct the deficiencies of traditional mathematics curricula. Afterwards, as test scores declined, they were less enthusiastic about the innovations. Some authors tried to "modernize" the traditional syllabus just by introducing a chapter on set theory. Also, some teachers were simply enchanted by the new ideas to the extent that they emphasized them too much. Furthermore, it was also the case that many teachers did not clearly understand the new content themselves and, therefore, could not teach it. All these situations contributed to the failure of the content before it had a chance to succeed.

The second point can be disputed. Kline's criticism was directed here against a deductive approach. He refered to the historical development of mathematics in which new inventions are usually derived by an inductive approach. Only later on - as the final step - is the mathematics systemized. and only here is a strict deductive way of proceeding used. But in the reform movement at the elementary level the pedagocical priciples take precedence over the logical ones. Children are supposed to discover mathematics for themselves through intuitive insight. At the secondary school level where tradionally the teaching of geometry was done deductively, content was reorganized to help students to perceive mathematics better through induction.

Furthermore, Kline maintained that pupils expect mathematics

to be presented as a "complete building" which can be applied to problems in their environment. This, however, is not as the well-known mathematicians themselves did, making great discoveries by experimenting and deducing.[1] It surely cannot be denied that on some occasions the deductive approach may have been somewhat overemphasized. Nevertheless, the importance of the logical approach as a fundamental, intellectual way of proceeding in mathematics must be acknowledged.

On the third point it may be noted that the role played by abstraction in the learning process is seen from different points of view. Kline accused the reformers of excessive abstracting beyond the children's level of development. This was especially true for the first phase, but during the second phase the child's learning capacity was considered, and the curriculum was redesigned accordingly. The starting point was the physical environment which became a base for later abstractions.

The fourth point criticized excessive introduction of symbols and new terminology. Understandably, in the early enthusiasm of organizing mathematics in syllabi, often symbols and terminology were introduced in a way which created more confusion than communication. For example, the differentiation between numbers and numerals could not be understood by many children or teachers. Studies were needed to determine which symbols were essential and which could be left out.

After this review of Kline's criticism one should not forget to take a critical look at test scores. As mentioned in the previous chapter, there were few suveys. For the three refered to there it was pointed out that the low scores were signs of a general decrease in performance. The results of the NAEP survey show that children taught with new materials were able to calculate, at least for upper levels. Thirteen and seventeen year-olds could calculate as well as adults. Their deficiencies have to be looked for in other fiels, and

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1) see CBMS (17) pp. 17 f.

these ought to serve as hints for new programs. The test scores do not give rise to the conclusion that poor achievements in mathematics are due to innovations.[1)]

The authors of the NACOME report arrive at the same conclusion.

> Taken together, the achievement data that NACOME has examined suggest two broad conclusions about the state of school mathematics achievement.
> 1) When it has been possible to compare similar traditional and modern mathematics texts, there has been a tendency for the traditional classes to perform better on computation while the modern classes perform better in comprehension.
> 2) There appears to have been a decline in basic scholastic skills since 1960. Mathematics achievement has shared in this decline. Recalling our observation in Chapter 1, that the reform movement in mathematics was not widely implemented in the classroom, there is still some evidence, that secondary classes using a modern program have tended to resist the general decline in mathematics. (CBMS (17) pp.118 f.)

Although the greater part of the arguments against the "new math" could be disproved and difficulties corrected, the public's reserved or even negative attitude toward the "new math" grew. This influenced the granting of money and in 1972/73 the financial support of the federal government decreased sharply. The following table provides a view of the effects on amounts obligated by NSF.

Table 2
COOPERATIVE COLLEGE-SCHOOL SCIENCE PROGRAMS (SECONDARY AND ELEMENTARY)
SUPPORTED BY THE NATIONAL SCIENCE FOUNDATION

Fiscal Year	Amounts Obligated	No. of Grants
1960	53 090	1
1963	744 130	14
1965	913 330	46
1967	2 296 295	56
1969	5 596 241	146
1971	4 898 741	158
1972	4 568 081	142
1973	2 376 256	81

(NSF (69) p.165)

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1) This view was confirmed to the author by the consultant in mathematics education of the State Department of Education in Sacramento, California, Dr. J. Hoffmann, who interpreted the results as an indication that new mathematics may have prevented an even greater decrease in achievement

The funding history of federal funds for special exemplary projects[1] by the National Diffusion Network[2] also shows the effects of changing attitudes during the middle seventies. However, this situation changed in the late seventies when new efforts arose to remedy existing deficiencies in education.

Table 3. Funding History, National Diffusion Network.

Fiscal Year	funds (millions)
1974	$ 9.1
1975	$ 8.4
1976	no funding
1977	$ 10.0
1978	$ 10.0
1979	$ 14.0
1980	$ 10.0

(NDN Reporter (65))

Another consequence of criticism was the closure of many Research and Development Centers and Educational Laboratories. In their place the National Institute Education (NIE) was opened in 1972. NIE, a central institute for research in education, became heir to USOE's competence gained in the R & D Centers and Educational Laboratories. NIE's tasks were to

1. Help solve or alleviate the problems of, and achieve the objective of, American education.
2. Advance the practice of education as an art, science, and professions.
3. Strengthen the scientific and technological foundations of education.
4. Build an effective education research and development system. (NIE (66) p.1)

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1) The term "exemplary" is conferred only after a project has been reviewed by the Joint Dissemination Review Panel of the Department of Education. This panel has responsibility for quality assurance, examining the evidence of both cognitive and affective evaluation presented by each project and judging its effectiveness. Positive endorsement by a majority of the attending Panel members constitutes approval, and a date of validation is assigned. (NDN (64))

2) The NDN is a nationwide system established to assist schools, postsecondary institutions and others to improve their education program through the adoption of already developed, rigorously evaluated, exemplary education projects.

within a short time grants for the NIE decreased as well.[1)]

Together with the criticism of the reform came the demand that schools should account for their practices in the manner of a manufacturer whose product is the children's learning achievement. This became known as the "Accountability Movement", which will be explained in the next section.

4.2. The Accountability Movement

In connection with the curriculum reform accountability refers to the schools' responsibility to students and financial supporters. Large sums had been invested in the reform, but there were scarcely any clues as to whether any goals of the new curricula were being reached. Recent technological developments have caused people to expect more effectiveness within all fields of human behavior particularly in the field of education. Because of this kind of expectation and doubts created by the critics of "modern mathematics" parents and taxpayers began to demand an accounting of the accomplishments of tax-supported projects.

In order to evaluate a project through student achievement the projects' goals must be described in terms of operationalized objectives and test items. Those efforts had become increasingly popular in the sixties as the behavioristic view of education grew. Research in these fields was advanced mainly by Gagné and Bloom who dealt with taxonomies of educational objectives.[2)] Mager, Glaser and Flanagan explored and defined methods for describing desired student behavior as measurable objectives. Efforts of this kind were overdue. The controls of measurable objectives and tests had been neglected in the preceeding years. But eventually, they were seen to be essential for evaluating and revising curricula.

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1) see Keitel (43) p.6
2) see pp.24 ff.

These efforts were one reason for the increasing focus on minimum competencies.[1] Formulation of objectives tends to restrict educational objects to those which can be measured easily, such as the mastery of computation skills. Objectives which could be measured only with great difficulty, such as the acquisition of attitutes and understanding of concepts, were easily neglected.

Another consequence of this emphasis on accountability in education was that schools tended to become "teaching factories" producing graduates of simple and identical competencies. Poor student achievement often was blamed on poor teaching. This is unfortunate because teaching itself depends on too many other factors. These may be seperated into individual prerequisites (intelligence, education etc.) and scholastic necessities (financial conditions of the school, social factors, attitudes of the responsible persons; other demands within the classes such as available materials, etc.). Clearly, neither the teachers nor the programs alone could be called to account for every aspect of students' performance.

A further consequence of this movement was that the only acceptable projects were those which oriented themselves to the behavioristic model and met the standards set by the legislatures or other groups that funded the programs. Projects such as the CSMP failed to meet those criteria and were threatened during the middle seventies.

These criteria were vigorously challenged by those mathematicians and mathematics teachers who feared the consequences of neglecting the teaching of concepts and attitudes toward mathematics.

> In an informal poll, reported in the November (1977) issue, 65 percent of the respondents felt that a more toward state assessment would result in an emphasis on low-level skills in mathematics at the expense of more important outcomes. (Minimum competence (51 a) p.85)

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1) see chapter 4.3.

Test results also showed that certain ethnic groups were having tremendous difficulties. Test scores of non-white children often were far below those of white children, especially in Florida and California. To counter this, legislative efforts were made to help these low achieving children by funding new types of programs such as the bilingual programs.

4.3. Minimum Competency

"Back to "Basics". This demand, often heard in the middle seventies and later, originated from two major causes:

1) Poor achievement results, especially in basic scholastic skills;
2) Accountability movement with its need for operationalized objectives.

Boards of education, legislators and citizens' groups called for programs that would guarantee the acquisition of basic skills. State legislatures started to require minimum competencies. By July 1976, 29 of the 50 states and District of Columbia had "performance-based educational activities",[1] by 1980 38 states required minimum competencies.

Although everybody was talking about basic skills there was little agreement of what these basic skills were. Many understood them to be the computational skills. However, restricting mathematics teaching to the teaching of just computational skills means going backwards to the curriculum preceding the reform movement. But mathematics teaching has become more than the teaching of only computational skills. Thus, efforts arose to find answers to the question, "What are basic mathematical skills?"

In October 1975 a "Conference on Basic Mathematical Skills and Learning" was held in Euclid, Ohio, under the sponsorship of the NIE. The invited participants were asked to respond

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1) see Taylor (87) p.88

in short papers to the following two questions:

1. What <u>are</u> basic mathematical skills and learning?
2. What are the major problems related to children's acquisition of basic mathematical skills and learning and what role should the NIE play in addressing these problems? (NIE (67) p.2)[1]

On the basis of these papers, the following four topics were suggested and working groups were formed around each:[2]

1. Classroom Instruction and Teacher Education
2. Curriculum Development and Implementation
3. Goals for Basic Mathematical Skills and Learning
4. Research Priorities.(NIE (67) p.3)

Working group (3) on goals for basic mathematical skills and learning developed three categories: "general goals", "basic goals", and "further desirable goals". Then three "general goals" were defined.

> Broadly, then, school mathematics should develop a student's ability to think ...
> Second, education in mathematics should encourage the ability to feel secure in situations calling for reasoning or quantitative thinking ...
> Third, mathematics should improve students' ability to <u>do</u> ... (NIE (67)p.17)

The "basic goals" are those "which should be attained by adults in our society. This is the mathematics needed by consumers, citizens and voters, and many workers."[3] Specifically, "basic goals" include the following topics:

- appropriate computational skills
- links between mathematical ideas and physical situations
- estimation and approximation
- organization and interpretation of numerical data, including using graphs
- measurement, including selection of relevant attributes, selection of degree of precision, selection of appropriate instruments, techniques of using measuring instruments, and techniques of conversion among units within a system

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1) These papers are published in NIE (66)
2) The reports of the working groups are published in NIE (67)
3) NIE (67) p.17

- alertness to reasonableness of results
- qualitative understanding of and drawing inferences from functions and rates of change
- notions of propability
- computer uses: capabilities and limitations (gained through direct experience)
- problem solving (NIE (67) pp.17-20)

By "further desirable goals" the working group understood those that "meet the greater needs of students whose interest in mathematics goes beyond the ordinary, either in training for a profession to which mathematics is basic or purely for its own sake."[1] The following topics were included in this category:

- recognition that mathematics is a construct
- ability to reason abstractly
- enrichment of the student's world
- accuaintance with the natural notations of mathematics
- mathematical modeling. (NIE (67) pp.20 f.)

Another answer to the question, "What are basic mathematical skills?" came from the Basic Skills Task Force of the National Council of Supervisors of Mathematics (NCSM) in its "Position Paper on Basic Mathematical Skills" presented during its 1976 Annual Meeting in Atlanta, Georgia. It also represented an expanded definition of "basic skills", that is, basic skills should involve more than just computation. They should include the requirements of the present technological society and such skills as "estimating, problem solving, interpreting data, measuring, predicting, and applying mathematics to everyday situations." [2]

The NCSM position paper lists ten basic skill areas:

1. problem solving
2. applying mathematics to everyday situations
3. alertness to the reasonableness of results
4. estimation and approximation
5. appropriate computational skills
6. geometry
7. measurement
8. reading, interpreting, and constructing tables, charts and graphs

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1) NIE (67) p.17
2) NCSM (56) p.148

9. using mathematics to predict
10. computer literacy. (NCSM (56) pp.148 ff.)

Of all these basic skill areas, problem solving is regarded as the most important. NCSM even claimed problem solving to be the "principle reason for studying mathematics."[1] This supported the working group on goals for basic mathematical skills and learning at the Euclid conference, who stressed the interrelationship of problem solving with all the other areas in mathematics. The same idea is expressed in the Addendum to the California Mathematics Framework in which it[2] serves as an "umbrella over all the remaining strands".[3] The importance of problem solving has been increasing since. In NCTM's Agenda for Action the first recommendation is that "Problem solving must be the focus of school mathematics in the 1980s".[4]

There would be an attractive reason for defining basic skills narrowly. Restricting mathematics instruction to computational skills would lead back to simpler days before the 1950s with much rote learning and a little mathematical understanding. But today's society cannot tolerate such simple solutions. Technology and other forces have created a new society that demands many more skills such as the basic skill areas recommended by NCSM and the working group on goals for basic mathematical skills and learning. Beyond that, the improvements in mathematics instruction with regard to the learning of concepts and mathematical understanding cannot be forgotten. The new approaches to learning together with new ways of organizing the content, though not perfect, have set mathematics education in a direction that should not be reversed. As NCSM states in its position statement on basic skills, "Yes, let us stress basics, but let us stres them in the context of mathematics instruction."[5]

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1) NCSM (56) p.148
2) problems solving/application strand
3) California Mathematics Framework, Addendum (10) p.3
4) NCTM (57) p.2
5) NCSM (56) p.147

4.4. Impact of Calculators and Computers on Mathematics Instruction

Technological developments in the field of computers and calculators have rapidly increased in recent years. The number of computers in use in 1975 was about 220,000. Together with this growing number of computers, a growing number of people has become involved in using those computers. The distribution of calculators grows even more rapidly and by 1980 about 80 million calculators were in use in the United States.[1] These drastic changes influence schools increasingly. This is especially true since the prices for calculators and computers steadily decrease.

Several arguments for the inclusion of calculators in school mathematics curricula were put forward by Max S. Bell, who was the chairperson of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics in Arlington, Va., in 1976:[2]

> 1. Calculators represent a kind of order-of-magnitude change in technology that can substantially change society.
> 2. One aspect of this change in society is that people outside of school may soon use only calculators to accomplis what we now make the principal component of years of schooling.
> 3. For schools to ignore this challenge poses unacceptable risks to the proper mathematical education of youngsters.
> 4. In fact, calculators have already had a considerable impact on education in and out of school. That impact is likely to increase except in the calculation curriculum, which may be very resistant. (Bell (5) p.410)

Many reasons may be given for the use of calculators in classrooms:

1) The decimal numbers found on the calculators should encourage introduction of metric measuring units.

2) Operations with decimal numbers are practised.

3) The computation is essential for application and problem-solving. Many true-to-life problems can be included which

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1) see NIE/NSF (68) p.1
2) see NIE/NSF (68)

would have been left out because of difficult calculations. Besides, for many low-achievers this always creates big problems, and through the use of calculators these weaknesses can be compensated.

4) Furthermore, by using calculators for routine calculations more time may be spent on the comprehensive and interesting tasks. The student gets more time for analysing the situation, for finding a solution and doing the mathematical operations as well as for evaluating the results.

Besides these mathematical arguments, there were others based on general education considerations.

1) Working with a calculator motivates the students.

   The calculator is generating interest in learning mathematics because computation can be simplified, therefore, providing more time to expand and reinforce the concepts. The student is becoming more inquisitive, creative, and independent as he experiments with the calculator and mathematics ideas. The recreational aspects using the calculator is very challenging at any level regardless of the amount of mathematics skill the students have obtained.

2) Calculators aid discovery.

   The calculator can be a unique aid in investigating concepts and problems that are new to the students. Using the calculator, students discover relationships and develop concepts that involve tedious or difficult computations.

3) Calculators vary drill and practice.

   The calculator is an excellent instructional aid for varying the routine nature of drill and practice. The essential skills of estimation and checking answers for their reasonableness can be practice using the calculator.

4) Calculators enrich curricula.

   The calculator is an extremely important aid in individual exploration and extension of the curriculum because it provides nonstructured enrichment opportunities through games and problem solving activities. (California Mathematics Framework, Addendum (10) p.23)

The question of how to include calculators in mathematics curricula needs still to be answered. Of the twenty-two recommendations of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics, seven deal specifically with curriculum development:

B. Curriculum development for the immediate future

Recommendation 6. Materials should be developed to exploit the calculator as a teaching tool at every point in the curriculum to test a variety of ideas and possibilities pending emergence of calculator-integrated curricula.
Recommendation 7. Research and development should be undertaken with respect to algorithmic processes in school mathematics, especially those in arithmetic and algebra.
Recommendation 8. Curriculum materials should be developed for kindergarten through twelfth grade to teach estimation, approximation, significant digits, order of magnitude calculations, and similar ideas.
Recommendation 9. Curriculum materials should be developed that teach problem-solving strategies more effectively and that build pupils' confidence in their ability to solve problems.
Recommendation 10. Curriculum materials should be developed and tested on new topics that are not now in the curriculum but that calculators may make feasible. The possibilities of adapting "advanced" topics to earlier levels should also be explored.

Curriculum development for the long-range future.

Recommendation 11. Some full-scale alternatives to the K-6 elementary school mathematics program should be developed that use calculators wherever appropriate (but not exclusively) and that considerably broaden the range of mathematical ideas absorbed during the elementary school program.
Recommendation 12. Some new courses for secondary school students should be developed, indluding consumer-industrial and data-oriented statistics courses, as well as alternatives for junior high school work and for those high school students who do not elect standard "college preparatory" mathematics courses. (NIE/NSF (68) pp.18 f.)

Also criticism against the use of calculators has been put forward. A major argument is that many students are not able to do the simplest computations and become too dependent on calculators. To avoid this the special skills of "number

sense", estimation skills, and understanding the decimal number system, should be required of everystudent. The Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics proposed the following skills and abilities:

a. Ability to multiply and add one-digit numbers nearly automatically.
b. Firm understanding of our standard numeration system. including reading and writing arbitrarily large or small whole numbers and decimals.
c. Firm understanding of the various meanings of fraction notation (e.g., parts of a whole, ratios, division) and ability to exploit "equivalence" of fractions.
d. Experience with algorithms. (NIE/NSF (68) p.13)

Much research needs to be done in this field. NACOME listed many questions that are still awaiting an answer:

When and how should calculator use be introduced so that it does not block needed student understanding and skill in arithmetic operations and algorithms?
Will ready access to calculators facilitate or discourage student memory of basic facts?
For which mathematical procedures is practice with step-by-step paper and pencil calculation essential to thorough understanding and retention?
What types of calculator design -- machine logic and display -- are optimal for various school uses?
What special types of curricular materials are needed to exploit the classroom impact of calculators?
How does calculator availability effect instructional emphasis, curriculum organization, and student learning styles in higher level secondary mathematics subjects like algebra, geometry, trigonometry, and calculus? (CBMS (17) pp.42 f.)

Computers also are having an increasing impact on school mathematics. Computers and related technology are used in school mathematics the following ways:

1. To present unfamiliar concepts and principles in an interactive textbook format to students at computer terminals.
2. To generate, administer, and score tests; to keep records; to assist teachers with a variety of other ancillary activities in teaching.
3. To provide individualized tutoring to students.

4. To drill students on new skills and help them practise these skills.
5. To diagnose learning problems and error patterns and provide remedial instruction to children.
6. To help students learn algorithms for solving many types of exercises and problems.
7. To provide students with a means of exploring new concepts and principles and in some instances to promote learning through discovery.
8. To manage complex, multimedia learning environment.
9. To give students an additional measure of motivation to learn mathematics. (Bell (4) pp.429 f.)

These nine points may be summarized in three categories:

- computer enhanced instruction
- computer-assisted instruction
- computer-managed instruction.

Computer enhanced instruction refers to the use of computers in the classroom as an additional means of instruction. The main goal of this category is computer literacy; that is, becoming acquainted with the ways computers work. Computer literacy can be found among the basic skill areas as set up by both the working group on goals for basic mathematical skills and learning at the Conference on Basic Mathematical Skills and Learning and the Basic Skills Task Force of NCSM.[1] Two other goals of computer enhanced instruction are

- to help motivate students to learn mathematics
- as an aid in learning problem solving skills.[2]

Computer-assisted instruction has already been described in section 3.5. Its influence on education has grown since the sixties and will continue to do so as technology expands and the costs of such products diminish. It provides individualization and is often used in programs aimed at special student populations, for example, remedial programs.[3]

The term computer-managed instruction refers to the use of computers

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1) see chapter 4.3.
2) see for further comments Bell (4) pp.431 ff.
3) see chapter 4.5.

- to adjust curricula
- to check tests and report the results
- to keep record of the students' work.

In this application the computer does not provide instruction nor evaluation of student performance as it does in computer-assisted instruction, but is a classroom management aid.. Such systems can be used with a wide variety of curricula whether or not it is based on measurable objectives and goals.

4.5. Accountability Conscious, Delivery System-Oriented Programs

The term "accountability conscious, delivery system-oriented program" has been used by B. Kaufman and N. Sterling for those programs which originate "in the concern with the methods of delivering content to children and with measuring precisely how well the content is digested by the children coupled with the growing disenchantment of professionals and nonprofessionals alike with the results of the 'new math'".[1)] There are a number of these programs, and they all make extensive use of behavioral educational psychology. Their main characteristics are

- a basis of a carefully constructed sequence of behavioral goals (complete with measuring devices to test for achievement of those goals) around which the content is organized;
- alternative learning activities based on individual achievement on diagnostic or placement tests.

They all operate under similar modes:

1) The teacher selects a set of performance objectives;
2) The student's needs are ascertained by means of tests (diagnosis);
3) Activities are prescribed according to the student's needs;
4) The student's achievements are evaluated.

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1) B. Kaufman, N. Sterling (41) p.1

The suggestions for activities and methods are many: classroom teaching, small group instruction, individualized instruction and math lab activities are only a few of the common ones.

The test problems are usually in the multiple choice form, and the answers may be marked on cards which are fed into a computer. The computer evaluates them and provides the teacher with exact and thorough test results by class, by students and by needs. The whole record keeping system may be managed by computer and, indeed, the computer can prescibe learning activities for students, and track students' individual progress.[1)]

Many of these programs are called diagnostic/prescriptive or remedial programs. They regard the child as a person who "suffers" from a lack of something. The first thing then is to determine these deficiencies through diagnosis. The next thing is to "cure" them by prescribing special methods and activities. These programs are especially designed for low achievers or socially disadvantaged students to be used as supplementary programs.

These general statements apply to four exemplary programs[2)] which shall be described in more detail as examples of this group of programs.

(1) Conceptually Oriented Mathematics Program (COMP)

The Conceptually Oriented Mathematics Program started developing materials for grades one through eight in the Columbia, Missouri, public schools in 1971. It was originally funded by the Office of Education under ESEA Title III. In December, 1974 COMP was approved by the Joint Dissemination Review Panel (JDRP) of USOE. Program Director is Mrs. Alta M. Harness.

COMP describes itself as "a management system that provides a list of sequential skills to be mastered in each of the basic skills areas. It is designed to meet individual needs

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1) see section 4.4.

2) see p.97

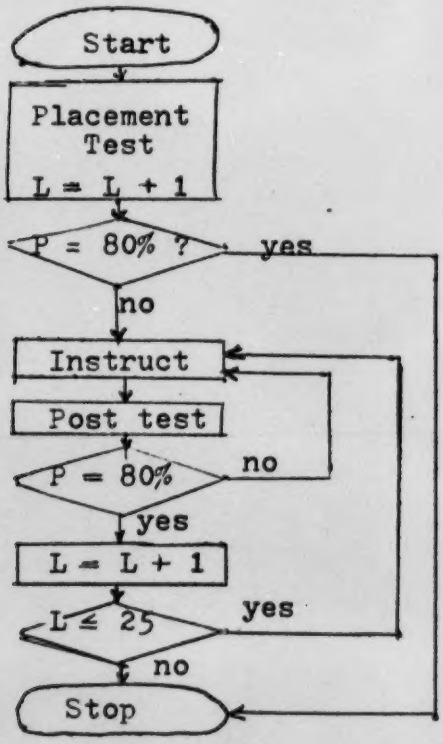
through small group instruction."[1] It has the following descriptors:

- Mathematics (Grades 1 - 8)
- Diagnostic/Prescriptive
- Management
- Crossgrade grouping
- Continuous progress
- Basic skills
- Performance based.

There are ten "basic skill areas" which are developed through twenty-five levels of complexity:

- sets
- numerals
- order
- addition
- subtraction
- multiplication
- division
- functions and graphs
- geometry
- measurement. (COMP (16) p.1)

The program operation is best described by the following flow-chart:



P = Percent of Proficiency
L = Level of Instruction

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1) NDN (64) p.9-18

To support the teacher the following materials were developed:

- Scope and Sequence Chart
- Teacher's Guide Book
- Placement Tests
- Post Test I
- Post Test II
- Individual Record Sheets
- Summary Levels for Parents (COMP (16) pp.1 f.)

By September 1976 COMP was being used in many Missouri private and public schools and has been widely adopted in Texas, New Jersey, and Alaska.[1)]

(2) STAMM: Systematic Teaching and Measuring Mathematics

STAMM is a mathematics program for kindergarten through high school. It started in the early seventies at Jefferson County schools, Lakewood, Colorado and received grants from the Department of Health, Education and Welfare and the Colorado Department of Education under USOE ESEA Title III. Program Director is Glyn H. Sharpe. STAMM's elementary program was approved by JDRP in June 1976.

STAMM's major objective is "to provide continuous process in mathematics for the entire school experience of all students, kindergarten through senior high school."[2)] The whole project is based on a framework of objectives and criterion/referenced assessment tests. "Careful monitoring of student progress, measurement of mathematics competencies, and alternative courses at the high school level are featured."[3)] The STAMM program includes a management system which helps in organizing the teaching of mathematics so that each student can be successful and challenged. However, no particular teaching style is dictated. The program may be used "in many different classroom situations including small group instruction, large-group instruction, individualized instruction, team teaching, and math lab."[4)]

The performance objectives cover twelve topics.

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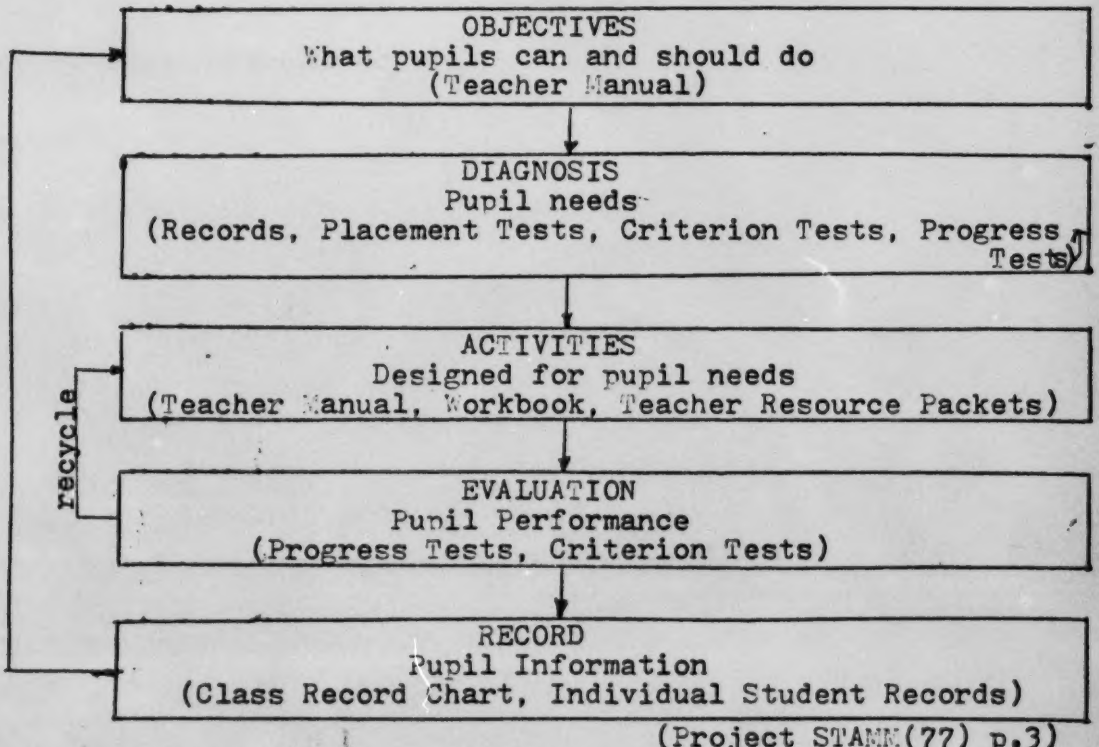
1) USOE (88)
2) NDN(64) p.9-67
3) NDN (64) p.9-67
4) NDN (64) p.9-67

- numbers and numeration
- properties and operations on numbers
- computation
- sets and logic
- equations and inequalities
- relations and functions
- probability and statistics
- graphing
- geometry
- measurement
- mathematical systems
- consumer mathematics. (Project STAMM (77) p.4)

All the topics in the program are covered at each of the grades K - 8. At the end of grade eight the students are tested on a set of 39 mathematics competencies. Students who show a deficiency can get additional instruction in the respective competencies.

Also, high school objectives have been developed for Algebra I, Geometry, Algebra II, Trigonometry, Applied Math, Vocational Math, and Rapid Calculation.[1)]

The teaching of STAMM works in the following way as shown in the model:



(Project STAMM(77) p.3)

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1) see NDN (64) p.9-67; Project STAMM (77) p.4

STAMM assessed student achievement at grades 4, 6 and 8 and received good results. Between two-thirds and three-fourths of all Jefferson County students scored above the national norm on the CTBS. This achievement has been consistent since 1973. Prior to implementation, only half of the students scored above the national norm.[1)]

(3) Diagnostic Prescriptive Arithmetic (DPA)

Diagnostic Prescriptive Arithmetic was originally developed in 1971 as a Title I program on Staten Island, New York. Since then DPA has become more than a remediation program and may be used in elementary grade classes as the arithmetic component of a mathematics program. In September 1974 is was selected by the USOE as an exemplary program. Program director is Mr. Matthew Scaffa.

DPA has as its goals

- to emphasize the use of diagnostic testing.
- to aid teachers to better meet the 'individual' needs of their students.
- to promote the use of concrete materials throughout the grades as a means of developing and modeling basic arithmetic concepts.
- to foster a more positive attitude toward mathematics among both students and teachers.
- to inspire teachers' confidence in their mathematical competency. (DPA (21) p.2)

As the name of the project implies DPA includes DIAGNOSIS using the Stanford Diagnostic Arithmetic Test for the identification of students' individual needs (conceptual weaknesses and skill deficiencies) and PRESCRIPTIONS to meet these needs in the form of activities and materials oriented to individual and small group instruction.

Activities and materials cover counting, place value, addition, subtraction, multiplication and division of whole numbers. The teacher's manual also includes suggestions for

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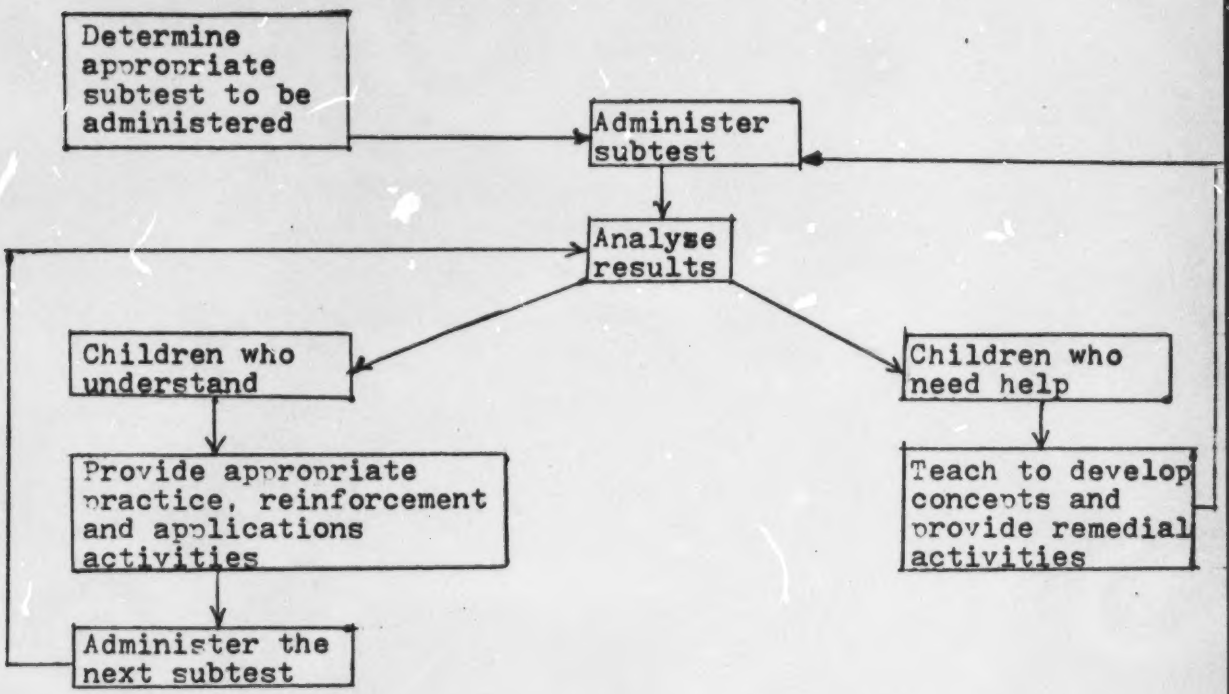
1) see NDN 964) p.9-67: Project STAMM (77) p.2

ongoing mathematics experiences in estimations, data collection, and applications to real life experiences.[1)]

The following diagram outlines the DPA process.


Determine appropriate subtest to be administered
Administer subtest
Analyse results
Children who understand
Children who need help
Provide appropriate practice, reinforcement and applications activities
Teach to develop concepts and provide remedial activities
Administer the next subtest

(DPA (21) p.5)

The target group of educationally and economically disadvantaged students achieved an average gain of 15 months of academic growth in mathematics for 7 months of instruction according to Title I criteria.[2)]

(4) Computer Assisted Diagnostic Prescriptive Program in Reading & Mathematics (CADPP)

The Computer Assisted Diagnostic Prescriptive Program started its work in Buckingham County Schools, Dillwyn, Virginia, in 1975 and was supported by USOE under Title I. The mathematics program developed by CADPP for grades 3 through 7 was approved by JDRP in June 1979. The program is directed by Debra Glowinski.

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1) NDN (64) p.9-23
2) see NDN (64) p.9-23; DPA (21) p.7

CADPP is a remedial program designed to supplement the regular school program. Its aim is "to allow for a usuable and accurate means of determining (diagnosing) a child's needs in basic ... mathematics skills, and to provide the teacher with materials and methods (prescriptions) to assist in teaching to the documented needs."[1)]

The program proceeds in the following way:

1) Diagnostic Testing(based on a locally developed standardized criterion-referenced test battery)
2) Individualized Educational Plans (provided) (IEP)[2)]
3) Prescriptions and Learning Centers

Special classrooms are set up with five learning centers or stations. Each station provides a different type of learning activities in the following sequence:

Teaching Station # 1 provides activities for the introduction of new concepts;

2, audio-visual station

3, games and manipulative station

4, independent activities

5, drill and practice.

Stations 2 through 5 are designed to reinforce skills utilized in the IEP.[3)]

> The participating students leave their regular program to receive ESEA Title I instruction within the facility for approximately 45 minutes a day, 5 days a week. Upon completion of their activity, the students return to their regular classroom. (CADPP (14) p.2)

4) Formative and Evaluative CRT testing.

The computer is a fundamental part of the CADPP operation. It is used to

1. prescribe learning activities for students based on individual achievement levels, learning modalities and special interests.

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1) CADPP (14) p.1
2) IEP is an individualized educational packet required by regulation in the Title I program.
3) CADPP (14) p.2

2. channel students to learning centers for teaching and reinforcing skill needs.
3. track individual progress so activities are not repetitive.
4. maintain continuous progress reports for students, teachers, and parents. (CADPP (14) p.1)

The first three programs offer from one to three days of teacher in-service programs prior to implementation. CADPP does not offer special workshops, but nonetheless, it requires extensive staff expertise in the design and development of criterion-referenced tests, instructional management systems, performance-process evaluation, monitoring, and individualized instruction via the learning station approach.[1)]

4.6. General Description of the Third Phase

The third phase, which saw the decline of the "New Math", is a period initially characterized by disappointment and decreasing enthusiasm. The predicted changes had not come true the way they had been expected. Instead, test scores were decreasing. Funding agencies started to force mathematics education away from the recent changes and towards"back to basics". Proficiency tests became obligatory to make the results of teaching available to anyone. Tougher standards were set up and had to be met. On this basis new programs tended to be reactions to these new demands.

Mathematics educators tried to find out the reasons for the deficiencies that had come to light. The problems of the past were analysed and new solutions proposed. Examples of the trends in the first half of the seventies can be found in the Snowmass Conference[2)], in the Tallahassee Conference[3)], and in the mathematics frameworks of California of 1972.[4)]

At the beginning of the California framework published in 1975 is a list of the new trends in the contents of mathematics instruction for grades K to 8.

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1) see NDN (64) p.9-17
2) see Report of the Conference on the K-12 Mathematics Curriculum (78)
3) see Proceedings of the Conference on the Future of Mathematical Education (75)
4) see Mathematics Framework for California Public Schools (49)

An increased emphasis on the application of mathematical concepts to physical objects familiar to children
An increased emphasis on computational skills along with the development of the structural aspects of mathematics
An increased emphasis on ways to improve children's attitudes toward mathematics
An increased emphasis on metric units known as the International System of Units, which will be the basis for standard measurement instruction
An increased emphasis on the total concepts of decimal numbers
An increased emphasis on application and problem solving skills
A decreased emphasis on numeration systems other than the familiar decimal-based system
A decreased emphasis on the computation of fractional numbers in kindergarten through grade six
A decreased emphasis on set theory. (Mathematics Framework for California Public Schools (50) p.1)

Another point can be added to this list, namely, the use of hand-held calculators in schools.

If one has a closer look at the areas of decreased emphases one will recognize them as some of the innovations during the 1960s. The teaching of numeration systems and set theory had been part of the emphasis on stressing mathematical structure. The teaching of computation of fractional numbers before grade six was part of the new organization of curricula necessar to add new topics such as modern algebra, probability theory and calculus. These innovations were now seen to be over-emphasized and it was the intention to overcome the mistakes of "new math" by redefining the curriculum. Numeration systems other than the decimal, for example, were only considered in so far as they could lead to a better understanding of the decimal system or to teach an awareness of the historical development of numeration systems.

A more comprehensive presentation of the problems and the trends of the third phase can be found in an article by H.-G. Steiner in 1976. He observed that the new efforts in mathematics should include the following:

(1) Investigating ways for furthering the problem-solving abilities of children, development of related teaching methods and materials

(2) Incorporation of meaningful (especially "real world") applications into the programs with special consideration of interdisciplinary or integrated curricula. Consideration of the special role of probability and statistics in these applications
(3) Investigating the place and role of computers and calculators in mathematical programs, especially in relation to problem solving and with respect to a meaningful use of calculators in arithmetic teaching in early grades
(4) Focus on Children. More emphasis on research in cognitive development and learning processes. Linkage between these studies and curriculum development efforts
(5) Identification and justification of goals and objectives of mathematical instruction for general education, reflecting the trend of an increasing number of students from all levels of ability and high school success to become "non-college bound". Find answers to the question: What would be a minimum residue for everyman from the school mathematics experience?
(6) Improving the flexibility of goals and means of evaluation and testing with special emphasis on creating better alternatives for integral and continuous component of curriculum development
(7) Better understanding of the role of teachers in the learning experiences of children and in curriculum changes and other innovative efforts. Needed analysis of the professional life of teachers, of pre-service and in-service teacher training practice, and development of ways to improve the teachers' professional competence. (Steiner (84) p.140)

Ever more the main goal of mathematics teaching became the optimal teaching of mathematics oriented at the basic mathematical needs of each child. Goals were first weight from the standpoint of the value which they could have for the general education of the child. This included an increasing consideration of those students who wanted to learn a vocation soon after leaving school. The first signs of this concern could already be noticed at the end of the second phase. In the third phase they appeared on a much larger scale, as articulated by one of the working groups of the Conference on Basic Mathematical Skills and Learning in 1975 and discussed in section 4.3.:

> The purpose of mathematics education is threefold: (1) to prepare the child for life as a consumer, voter, and citizen; (2) to begin the training for a variety of productive occupations and professions; and (3) to assist the child in developing a rich and rewarding life. (NIE (67) p.17)

So, it is no longer the education of future mathematicians, scientists and technicians, but mathematics instruction is to be oriented towards the needs and demands of an average citizen.

In this phase the reformers did not strive to reach such a high level of abstraction as they did in the beginning of the reform. The "new mathematics" had been accused of over-emphasizing mathematical structures and concepts as well as neglecting the acquisition of calculation skills, applications and problem solving strategies. Shifting the emphasis to the other extreme would bring back the state of mathematics education to what it was before the reform started. Therefore, the reformers looked for a synthesis of both aspects. The acquisition of calculation skills should go along with the development of structural aspects. In order to solve problems and apply mathematics, a working knowledge of mathematics was an indespensible prerequisite. The pure mathematical learning was to be limited to the point where insights into the basic structures and concepts seemed to be necessary prerequisites for the application and solving of problems.

As far as application is concerned, an inspection of textbooks and tests show a quite disappointing view of "application", namely, it often means just "word problems", frequently being restricted to a few simple situations. This trend was supported by the call for accountability. However, there have been some efforts to apply mathematics in a broader field and relate more closely with other subjects.[1)]

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1) see MINNEMAST, USMES, interdisciplinary curricula in section 3.7.

Recommendations on the interrelation of school mathematics and science during this phase have been presented in a number of curriculum planning conferences including the Cape Ann Conference on Junior High School Mathematics (1973), Snowmass Conference on K - 12 Mathematics Curriculum (1973), Estes Park Conference on Learning Through Investigation and Action on Real Problems in Secondary Schools (1973).[1] A project at Boston University producing seventh and eighth grade mathematics courses followed the lead of the Cape Ann Conference. It emphasized mathematical thinking in a context of applications especially with the following topics:

> 1. Orders of magnitude and estimation; 2. Functions, graphs, and their applications to life sciences as well as physical sciences; 3. Descriptive statistics; and 4. Space perception and representation. (CBMS (17) p.29)

Besides problem solving and applications, another new topic became increasingly important, the metric system as the standard for measuring. In 1975 the Metric Conversion Act passed by Congress. It declares

> that the policy of the United States is to coordinate and plan for the use of the metric system and to encourage educational agencies and institutions to prepare students to use the metric system of measurement as a part of regular instruction. (Moss (52) p.3)

Eventually, the President appointed a seventeen person U.S. Metric Board to facilitate the orderly transition to the metric system. Many states, such as California, have their own metric board or metric conversion council. The metric system is attravtive to mathematics educators because it is more closely connected with the decimal number system than were the old U.S. customary measuring units. For the mathematics curriculum metrics means familiarity with the use of meter/ kilogram/liter and the treatment of the metric prefixes which are based on the number 10 as in the numeration system.[2]

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1) CBMS (17) p.26
2) see CBMS (17) pp.43 f.

Among the exemplary programs of the NDN there are two metric programs, "Go Metric" and "Metrics Made Easy".

"Go Metrics" is designed to be a supplemental low-cost metric curriculum. It was developed and implemented in the Bismark School System, Bismark, North Dakota with Title III funds. It may be described in the following way:

> This innovative program includes an elementary and secondary curriculum for all pupils in the school population, and identifies a variety of teaching techniques involving the pupils in a variety of "hands-on" activities using metric equipment. Audio, visuals, and games are also utilized to accomodate the special needs of all students. To provide additional in-depth understanding of metrics, the inservice requires teachers to participate in the same metric exercises that are used in the classroom. The curriculum is arranged so that it does not intrude on an already crowded schedule, but enhances metric instruction as teachers integrate it into appropriate instructional areas. (NDN (64) p.9-32)

"Metrics Made Easy" has been developed in the Ocean View School District, Huntington Beach, California, through federal funds. It is described as a "systematic hands-on approach to metrication that includes field-tested techniques, activities, an assessment system, and staff development materials."[1)] Its three main goals are

1) to introduce and reinforce metric terminology
2) to provide practical measurement experiences using metric measurement instruments in the areas of length, mass, capacity and temperature and
3) to understand that a complete metrication program by the United States will include social, economic and scientific benefits. (Moss (52) p.2)

This program is primarily a teacher inservice program designed to bridge the gap between inservice and classroom application of metric activities.

When one considers all aspects of the curriculum projects of this phase, it is clear that there were no great changes in the methods of teaching in comparison to the previous phase.

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1) NDN (64) p.9-47

The projects still made use of individualized instruction, group instruction and classroom teaching. However, many mathematics teachers stuck to traditional classroom teaching practices including lectures, discussions, chalkboard work, teacher demonstrations and individual assignments.[1] Mathematics laboratories became more and more popular. On the other hand, the great discussion on "discovery learning" ceased. The schools' main interest became maximum results in achievement and proficiency tests at the costs of neglecting methods that aimed at discovering and enjoying mathematics. Mathematics laboratories are supposed to offer students numerous possibilities for discovering mathematics, but instead, students often are given tasks that do not encourage experimenting. Teachers push the student forward in preset directions instead of guiding him and using his ideas. Examples for this can be found in all the projects that provide lessons that are completely structured with programmed materials.

Research in mathematics education at this period concentrated on the child's cognitive development in conjunction with the learning process. Piaget's stages of development played an especially great role. Much research was also done on various methods of teaching such as self-paced mathematics instruction, small group instruction, and on the teaching of special areas such as problem solving and calculators.[2]

Two research projects shall be briefly mentioned.

1) Project for the Mathematical Development of Children (PMDC). This project, located at the Florida State University at Tallahassee, is directed by Eugene D. Nichols. Concentrating on children at the age of five to eight years, it has investigated the particular learning processes involved in the learning of mathematical concepts and skills.

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1) see Fey (33) p.495
2) see for abstracts and analyses of recent research: The Center for Science and Mathematics Education (13)

2) Problem Solving Strategies and the Use of Hand Calculators in the Elementary School

This is one of many projects that did research on the classroom use of calculators, giving special consideration to its effect on learning problem solving strategies. It was headed by John Kelly and Ira Lansing at Indiana University.

One solution to the widespread problems of how to provide in-service teacher education was the "Teachers Centers". These centers are

> Informations- und Kommunikationszentren für Lehrer - zum Teil von Lehrern geschaffen -, in denen Probleme aus der täglichen Arbeit, curriculare Fragen, Organisations- und Strukturentscheidungen vorgestellt und diskutiert werden können. (Gerbaulet and others (34) pp.55 f.)

The basic conception of these centers is the integration of a twofold purpose:

1) They should contribute to a closer cooperation among all the educational institutions in a given locale.

2) Their main task should not be the sponsoring of specific projects, but seeking comprehensive solutions to the problems facing local educators.

The main characteristic of these centers, therefore, is a connection between research and teacher-education. These centers are established and supported by a variety of agencies including individual school districts, states, and the federal government. They usually find their work in the specific needs of their own region. Their approaches to these needs include the following:

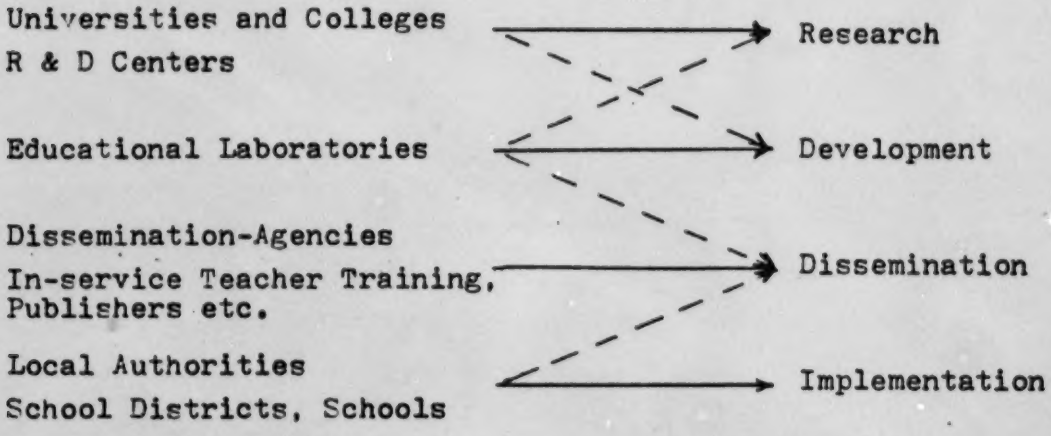
1. Um- und Neudefinition der grundlegenden Ziele der Schule.
2. Die Feststellung, inwieweit die bisherigen Ziele mit bereits vorhandenen Programmen eingelöst wurden.
3. Ermittlung bisher nicht erfüllter Bedürfnisse
4. Die Entwicklung alternativer Strategien zur Befriedigung dieser Bedürfnisse

5. Feedback an Entscheidungsträger, um eine Verbesserung der Schulentwicklung gemäß den identifizierten Prioritäten zu gewährleisten
6. Die Initiierung eines Arbeitsprozesses, durch den sichergestellt wird, daß durch die flexible Verbindung von Kritik aus der Praxis und Lösungsvorschläge für die Praxis eine 'rollende Reform' zustandekommt. (Gerbaulet and others (34) p.191)

These tasks may be divided into three main ones which are clearly interrelated:

1) research oriented at the needs of schools
2) pre-service and in-service teacher education
3) curriculum development.[1)]

Teachers centers offer an alternative to the usual "Engineering Model" for innovation strategy in education. The roles of the various institutions in translating research into implementation are shown for the Engineering Model in the following chart:



(Steiner (84) p.136)

In the Teachers centers alternative a closer cooperation among the various institutions and agencies is possible as well as closer attention to practice based on the needs of individual schools. Also, teachers now may play an active part in developing curricula. At the same time they would become familiar with the research in new contents and methods. The

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1) see Gerbaulet and others (34) p.191

Teachers centers supported the trend away from the monopoly held by universities and other places of higher learning providing on the content and the methods which should be used in teaching. In this phase, the projects started from the basis of the needs found in an individual school and then spread out to schools with similar needs.

Another main task of the centers is to offer courses for in-service teacher training. This training can be more effective than the usual summer courses because of the close cooperation between the schools and the centers. However, the Teachers centers did not really become popular. In-service teacher education remained a task of universities, state departments of education and similar institutions. There were no really new efforts.

Finally, some findings on changes in mathematical achievement, 1973 - 78, shall be examined closely. The National Assessment of Educational Progress surveyed the mathematics achievement of 9-, 13- and 17-year-old students a second time during 1977 - 78.[1)] The coverage of measured content as compared to the first survey was not as broad. Students' abilities were measured at four different cognitive levels crossing a variety of traditional mathematics content areas: (1) knowledge; (2) skills; (3) understanding, and (4) application. Major content areas were

> numbers and numeration; variables and relationships, geometry (size, shape and position); measurement, and "other topics", including probability and statistics, and graphs and tables. (NAEP (55) p.xi)

Testing conditions were as near as possible to the first assessment.[2)] Average performance on all mathematics items assessed showed a very slight decline in 9-year-olds' performance, a slightly larger decline for 13-year-olds and an even larger decline for 17-year-olds.

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1) compare first survey pp.88 ff,
2) see NAEP (55) p.xii

The Report found that the average performance on mathematics knowledge items (the first cognitive level) did not differ significantly from the 1973 results. However, for individual items the following changes are noted: substantial gains of 13- and 17-year-olds in knowledge of the metric system; declines of 13- and 17-year-olds in ordering fractions and decimal numbers; and knowledge of English unit relationships declined at all ages.

At the next level, the average performance on mathematical skill items showed no changes for the 9-year-olds' skill levels, but some decline in average skill levels of 13- and 17-year-olds. Whole number computational skills of 9- and 13-year-olds appear to have remained relatively stable, whereas 17-year-olds may have lost some grounds in their skills with more complex computations. The results for computation with fractions and per cents declined appreciably for all ages. Skills on measurement and on reading tables and charts did not show any significant changes for 9- and 13-year-olds. However, performance dropped on following items: skills on calculation with exponents and roots, measurement skills, reading tables and charts, algebraic expressions and equations of 17-year-olds; and skills on manipulating algebraic expressions of 13-year-olds.

The average performance on mathematical understanding showed slight declines for 13-year-olds and more drastic ones for 17-year-olds. Considering individual items, the following changes occur: improvement of 13-year-olds on items measuring translation between forms of mathematical expressions; a drastic decline in the understanding of number theory and in the properties of numbers and operations; a decline for 17-year-olds on understanding of geometric principles as well as on interpretation of information presented in graphs.

The average performance on the application items for all three age groups showed a significant decline. Declines were noted especially in the following areas: work problems involving multiplication and division, problems involving

fractions for 17-year-olds, multi-step problems, application of geometric concepts, items involving probability and statistics, and items using information from charts and tables to solve problems.

To understand these changes, many factors must be considered, such as societal changes, changes in education practices and changes in curricular emphases. These results are basically in accord with patterns found for other subject areas. NAEP asked a panel of persons active in mathematics education to comment on these data and consider possible implications. Keeping the recent changes in education (and especially changes in mathematics education) in mind, a number of factors were seen as contributing to these declines:

- emphasis on "back to basics", which often resulted in a narrowing of the curriculum, with more attention focused on computational skills and knowledge of facts and definitions and less time on problem solving;
- adoption of a simplified approach to problem solving by many textbooks;
- vocabulary difficulties in problems;
- mechanistic approach to problem solving;
- increased emphasis on testing resulting in emphasis on areas that are easy to teach and easy to test;
- devoting too much time on prerequisite lower-level skills and knowledge resulting in reducing the time available for problem solving;
- too little emphasis on probability and statistics.[1)]

Finally, a number of recommendations were made in light of these data, some of which were repeated in 1980 by the NCTM's "Agenda for Action. Recommendations for School Mathematics for the 1980s".[2)]

1. An expanded definition of what is "basic" in mathematics is crucial to foster students' ability to cope with different types of mathematical problems. Students must

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1) NAEP (55) pp.25 f.
2) see pp.148 f.

be introduced to exercises involving higher-level as well as lower-level cognitive processes.

2. Textbooks should be modified to include a greater variety of problem-solving tasks. If material is not included in textbooks, teachers may be less likely to introduce extra material that might not be used than to reduce the material included to the minimum.
3. Teaching of problem solving should receive more emphasis in the schools. It appears that many teachers feel outside pressure to drill for mastery of skills and not to teach problem solving.
4. Success on tests should not be the only criterion used to measure the effectiveness of mathematics programs. Too often the test then becomes the objective, and areas that require more time to teach or are not as easily subject to modification fall by the wayside.
5. Approaches to problem solving should not be of the type that encourage mechanistic, algorithmic methods. It should be emphasized that the ability to analyze a problem situation is equally as important as the correct solution.
6. Research on effective curricula and teaching approaches is needed to provide a clearer idea of strategies that will improve mathematics achievement.
7. Additional research into causes and potential remedies for disparities in groups' performances is needed. (NAEP (55) p.27)

5. GENERAL SURVEY OF THE ENTIRE REFORM

5.1. Main Issues

The detailed description of the three phases of the mathematics reform now shifts to an attempt to compare the major emphases and to see how each is related to the other. The issues are made clearer by means of summary tables which are based on only the projects described in this paper. In this respect it must be kept in mind that the projects described in this paper are already a selection.[1)] They are either well-known projects or representative of certain groups of projects, for example, those for low-achievers. However, there exist an indefinite number of projects which time and space have prohibited mentioning in this paper.[2)]

Table 1 charts the targeted groups of students for the projects over time. It shows how the reform started in the upper grades for talented students. This emphasis is compatible with the general goals of this phase, namely, to provide better preparation in mathematics for college-capable students. After 1958 another goal was added to train qualified mathematicians, scientists and technicians in a technological competition with the USSR. Here, again, these endeavors concentrated on the average and the above average talented students. There were few attempts to develop curricula for low-achievers and even then, these attempts were based on the view that these students were able to learn the same things as the able students if they were given more time. The common objective was to assist all students in reaching the highest level of mathematics possible. The second phase begins when the importance of a solid mathematics foundation in elementary school on the entire mathematics curriculum was finally noted. At first efforts were again restricted to the able pupils. Only in the latter half of the sixties did the reformers turn towards low-achievers on a larger

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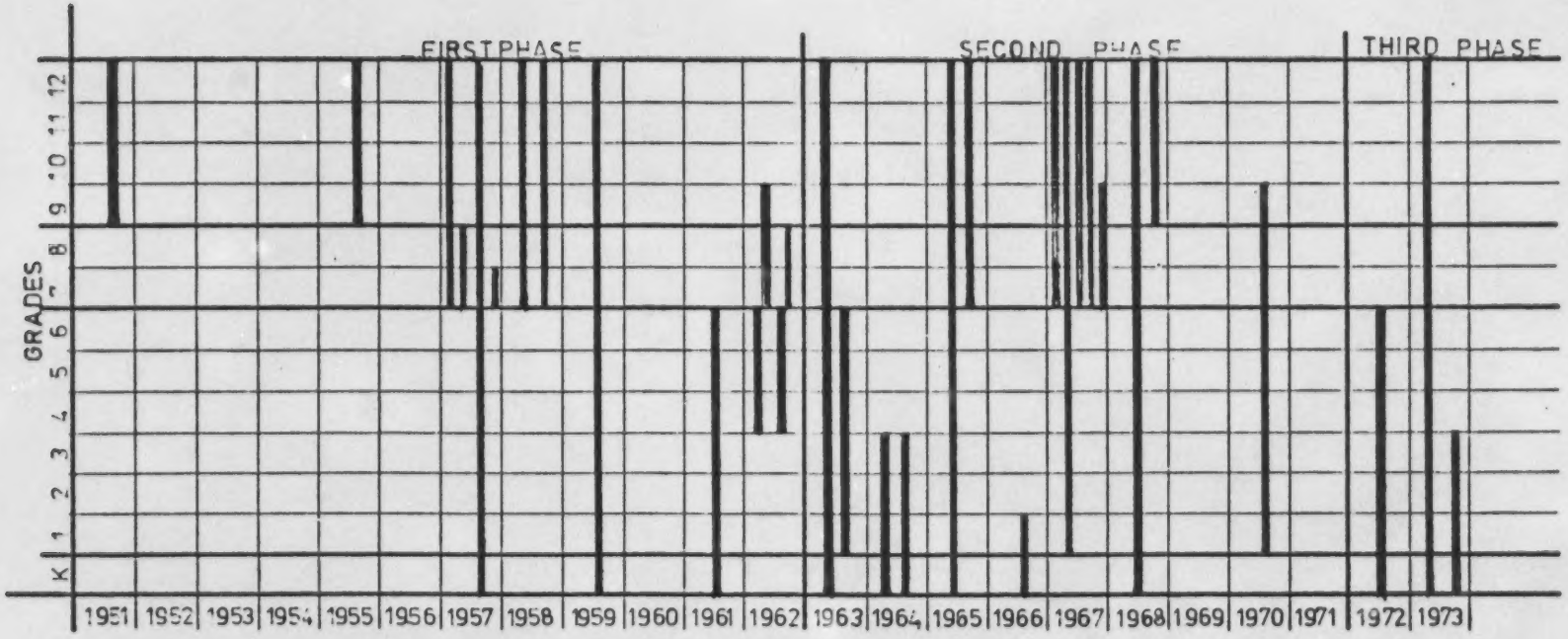
1) for a complete list see pp.150 ff.
2) see section 3.4.

Table 1

Targeted Students in the Main Reform Projects over Time



red - above average talented students
blue - average talented students
green - low-achievers

scale. The representative projects selected for discussion in this paper can only give a hint of this tendency, and there were actually far more of them than can be mentioned.

The projects of the second phase also clearly reflected the dominant goals of this phase. First, attention was to be given to all pupils, and second to provide a thorough education commensurate with each pupil's level of achievement. Table 1 shows that the two tendencies which came up in the second phase expanded further in the third phase. In addition to this, many remedial programs appeared in the 1970s.

Table 2 shows the content and the methodological trends in the reform. The tendencies are grouped into four conceptions which provide the theoretical basis for the various projects described in this paper. Again it should be noted, that the curves can only show the general trends because it would be nearly impossible to include an account of all the projects.

The main concept which can be found during the first and the second phases is what Keitel called "New Math".[1] This conception concentrates on a reform of the content of mathematical instruction.

> Ausgangspunkt dieses Konzeptes ist die Systematisierung der Wissenschaft Mathematik als Strukturmathematik durch die Gruppe Bourbaki, ihr Neuaufbau in einheitlicher Sprache und präzisere Begriffsbildung.
> (Keitel (43) p.18)

This conception could not tolerate the organization and presentation of school mathematics that existed of that time because it did not satisfy the new demands for precision and uniformity of language and concepts. On the elementary school level the traditional topics remained generally the same. The number concepts was defined in terms of sets. On the secondary school level new topics were

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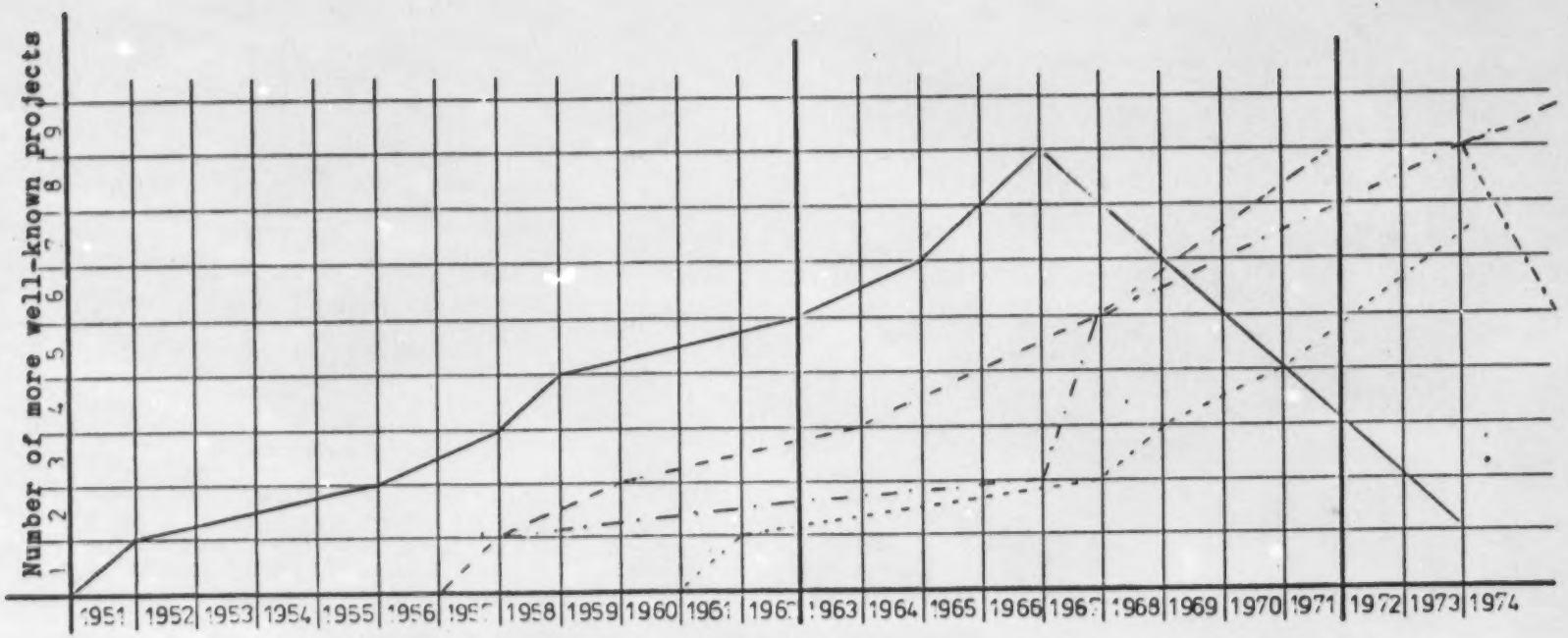
1) see Keitel (43) pp.18-20

Table 2

Trends in Curricular Conceptions for the Main Reform Projects over Time



——— - "New Math" conception
– – – - behavioristic conception
– . – . - structural and functional conception
..... - interdisciplinary conception

introduced such as probability theory, statistics, computer mathematics, and calculus and a deductive approach was used for treating these topics. The application problems were pushed to the sidelines. It was regarded as more important to understand the basic mathematical concepts and structures than to acquire simple calculation skills. This conception clearly shows the emphasis of the first phase, namely, the stress on abstraction, orientation to college mathematics and neglect of methodological aspects. The projects which follow the "New Math" conception are the UICSM, the Commission on Mathematics, the SMSG, the Cambridge Conference and the BCMI.

The behavioristic conception had not yet been accounted for in terms of content and methods. The projects based on this conception came up for the first time during the second half of the first phase. They were independent with respect to content and, therefore, could include any of the topics from the "New Math". The behavioristic concept strives for well-organized, always verifiable teaching. The theoretical foundations can be found in behavioral science. This science starts from the assumption that all learning can be described in terms of stimulus-response connections. According to this view the educational objectives have to be set up in such a way that the response conditions are defined precisely. The objectives are written so that they can always be verified. Thus the main task is the formulation of such objectives. The projects based on this conception became quite popular in the second half of the second phase and increasingly popular in the third phase. Because of their precise construction they served as the basis for programs using computer assisted instruction. These programs started at the pupils' individual achievement levels from which the objectives were determined. Then problems were set up which had to be solved by the pupils in order to reach those objectives. This kind of project strives for optimal individualization. Even so,

this type of project demands a certain restriction on the behavior, disregards the learners' behavior to a certain extent, and forces him to respond with a certain kind of predetermined behavior, in spite of all the attempts to consider the child's individual cognitive and affective position. Advantages and disadvantages were already mentioned in sections 3.5. and 4.2. Some of the projects which basically follow a behavioristic conception are the UMMaP, the GCMP, the Stanford Project, Project PLAN, Project LOCAL, and those described in section 4.5.

The third group of projects consists of those which Christine Keitel divides into the categories "structural" and "functional".[1)] Both types of projects concentrate partly or exclusively on the methodological aspect. Both take into account psychological findings on the cognitive processes as well as personality characteristics such as creative abilities, achievement motivation and factors which determine intellectual behavior.

> Das strukturalistische Konzept basiert auf Untersuchungen von kognitiv orientierten Lerntheoretikern zu Begriffsbildungsprozessen, aufgrund derer Jerome Bruner seine Theorie "Structures of the Disciplines" entwickelte. Diese Theorie beinhaltet in schematisierender Verkürzung den folgenden Gedankengang: Die kognitiven Strukturen sind die jeweiligen Gesamtheiten der erworbenen Begriffe und Denkfähigkeiten. Sie entwickeln sich von einfachen, aus wenigen Begriffen gebildeten Strukturen durch Hinzugewinn neuer Begriffe zu immer entwickelteren kognitiven Strukturen. In ihrer höchst entwickelten Form entsprechen die kognitiven Strukturen den Strukturen der Wissenschaften, die als die Essenz aller in ihr enthaltenen Begriffe und Prozesse verstanden werden. Diese Strukturen sind so komplex, daß sie alle prinzipiellen Einsichten, Begriffe und Verfahren der Wissenschaften umfassen, und zugleich so einfach zu formulieren, daß sie auch einer niederen Stufe der Erkenntnis vermittelbar sind. (Keitel (43) pp.20-21)

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1) see Keitel (43) pp.20-25

The purpose of education in this theory is to transmit the scientific concepts to the pupils with a simpler structure of knowledge.[1] In order to reach this aim, Bruner advocates the development of decision making processes through the discovery learning.

> Hauptaufgabe der Curriculumkonstruktion ist es, für diese Entdeckungsprozesse sinnvolle und anregende didaktische Modelle als Verkörperungen von Strukturen zu erfinden. (Keitel (43) p.22)

The topics must be presented according to Bruner's "spiral approach".

The functional conception starts from two assumptions:

> zum einen davon, daß alle schulische Erziehung abzielt auf eine optimale Grundausstattung des Schülers mit kognitiven, affektiven und motivationalen Fähigkeiten; zum anderen, daß diese Fähigkeiten als Faktoren von Persönlichkeitsmerkmalen zu beschreiben sind, als welche sie von Psychologie, Sozialisationstheorie oder Intelligenztheorie identifiziert werden. (Keitel (43) p.23)

Piaget's investigations and his theory can also be adapted to this conception.

Some of these projects could be found in the first phase but most of them appeared in the second phase. In the third phase nearly all the projects gave way to those based on a behavioristic conception. With the rise of these structural and functional concepts the methodological aspect was increasingly stressed. These projects are clearly detached from the "New Math" conception. The projects in this category are the Madison Project, CSMP and those which were described for low-achievers in the second phase.

The last group consists of interdisciplinary projects, which arose rather late in the progression of the reform. This interdisciplinary conception

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1) Keitel (43) p.21

> entsteht gleichzeitig und auf der gleichen kognitionstheoretischen Basis wie das funktionalistische Konzept, versucht aber, über die aus diesem Ansatz zunächst folgende Beschränkung auf Aussagen zur Methodik hinaus auch zu inhaltlichen Bestimmungen zu gelangen. (Keitel (43) pp.25 f.)

Also the selection of topics ought to be oriented at the students' needs and interests. For this situation it is often necessary to loosen the borderlines between subjects so that a problem can be solved by concepts and processes derived from several disciplines. Special emphasis is placed upon the student motivation. The projects which fall into this group are MINNEMAST and those described in section 3.7.

In summary, the more the reform progressed the more the methodological aspect was taken into account, and so were curricula oriented at the needs of the child. This reform of the methodological aspect includes among others, the advancement of discovery learning, the use of manipulative materials, the establishment of mathematics laboratories and the use of a variety of methods and media. At first the scientific interests and precision of language were stressed. The result was that the content side was overemphasized and the question of if or how students could reach the established aims was neglected. The content of that period was characterized by the abstractness characteristics of college mathematics, and the acquisition of computation skills was discredited along with concrete applications. However, the importance of these last topics increased in the course of the reform and today are among the most important ones. Now whenever possible the reformers interweave the applied aspects together with the abstract mathematics and they continue to aim at the understanding of mathematical concepts and structures. Another shift in emphasis could be noticed in the source of the development of curricula. In the first phase and in the main part of the second phase this was the task of university professors. They tried to reform teaching from "above". The schools and

the teachers just presented the curricula as they were handed down from the universities. Today, there is an attempt to start from the "grass roots", to bring new ideas out of the school milieu and from those students for which the respective curricula are intended. Necessarily, the work is done by the teachers themselves. A survey of the shifts in emphases is given in Table 3.

5.2. Results

The reform was a large-scale attempt to innovate goals, contents and methods of mathematics teaching in the schools using the latest scientific research, expert opinions, and the needs of society. The scientists of all the participating disciplines strived to develop carefully planned curricula which fully met the shortcomings of the traditional curricula. Some of these deficiencies were the emphasis on the acquisition of skills by meaningless drills, the accompanying neglect of understanding, out of date content, disconnected treatment of topics in geometry and algebra, poor motivation, and neglect of important topics associated with needs of society. It was simply hoped that better results in mathematical instruction could be attained by forcing students to work harder.

To what extent did the "New Math" influence schools? Although the reform movement had been in progress for more than two decades it did not reach all schools and communities. One clue about the size of the impact of the reform can be seen from the geographical distribution of its projects. Except for a few projects of national scope such as the SMSG, even the bigger projects had been limited to certain areas where their materials were developed and tested. On the map one can see the locations of the reform projects, and where the reform most likely reached the schools. Most of these regions are densely populated, particularly the east cost, the area around the Great Lakes and California. Even so, it was usually not one

Table 3

Shifts in Emphases

Reform from "above" (universities)	⟶	Reform from the "ground level" (Teachers' Centers, schools)
Stress on the content side	⟹	Increasing stress on methods, but without neglecting content
High degree of abstractness	⟹	Increasing stress on concrete situations and materials, applications and problem solving strategies
Main accent on understanding of mathematical structures and concepts	⟹	Acquisition of skills together with acquisition of concepts
Curricula for average and above average capable students	⟹	Curricula for all levels of achievement
Curricula for upper classes (junior and senior high school)	⟶	Curricula for all grades and ages
Mathematics as preparation for the training of qualified scientists and technicians	⟹	Mathematics as necessary all-round knowledge of every citizen and as preparation for all professions
Selection of goals by a few "experts"	⟹	Stress on achieving standards in proficiency tests imposed by legislature

Geographical distribution

of projects

red - projects of the first phase
blue - projects of the second phase
green - projects of the third phase

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of the projects' major goals to have a direct effect on schools on a large scale. The textbooks were the major medium for disseminating the new trends in school mathematics. The message they carried, however, represented "New Math" in quite different ways. Many of them gave the appearance of being rather "modern". They might offer set terminology, various numeration systems, or an introduction to new topics such as probability and statistics, but the changes in content ended there. Topics such as geometry were treated like they had been for many years. Other books overemphasized structure and concepts and were used by few schools.

In spite of the criticism above, there have been many improvements in textbook content and presentations as a result of extensive revision of the materials. In working over the textbooks, considerable attention was given to motivation, relevance to the child's world, and pursuit of higher mathematical standards. Also, many small projects that were developed to meet special needs of a school or school district contributed to the change in school mathematics.

Even where "New Math" reached a school, there are several factors that explain the different degrees to which a school was affected. A great deal depended on the mathematics teachers themselves, the extent they were willing to be open to the reform how enthusiastic they were about it and their involvement in professional teacher associations that publicized the reform movement. In this connection the NCTM should be mentioned as a national association of mathematics teachers which contributed to the reform in many ways: serving as a forum for new ideas; publishing many articles contributing to the reform in their magazines, for example in the "Mathematics Teacher" and "Arithmetic Teacher, or even books[1]; dedicating several of

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1) NCTM. The Continuing Revolution in Mathematics (58)

of their yearbooks to those topics which were concerned with the reform[1]; appointing its own committees, for example, the Secondary-School Curriculum Committee that published its report in May 1959[2]; funded and/or designed its own surveys and projects.[3]

Another factor that effected the influence of "New Math" can be surely seen in the structure of school administration in the USA. There are few curricula mandates at the state level and none at the national.[4] Most of the schools or school districts can decide for themselves which contents and methods they want to use. This fact is related closely to the way schools are financed. Innovations can only be introduced when there is enough money. The financial state of schools varies greatly among the state and even the school districts within each state. Innovations can be introduced in poor regions in a very limited range, if at all.

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1) NCTM: Insights into Modern Mathematics, Twenty-third Yearbook (61)
NCTM: The Growth of Mathematical Ideas, Grades K-12, Twenty-Fourth Yearbook (59)
NCTM: A History of Mathematics Education in the US and Canada, Thirty-Second Yearbook (60)

2) NCTM: Secondary Curriculum Committee. "The Secondary Mathematics Curriculum" (63)

3) see CBMS (17) p.68
see PRISM: Shedding light on NCTM's Recommendations for the 1980s (76)

4) An exception is the State of New York. The New York Board of Regents developed exact curricula for schools, and it monitors the schools to determine if they are meeting the curriculum requirements.
More typical is the state advisory function as in California which has published three mathematics frameworks since 1963, the purpose of which is "to provide a base from which schools school districts, and offices of county superintendents of schools can develop adequate goals and objectives for their programs. In addition, the mathematics framework provides the basis for the development of criteria for the evaluation of instructional materials to be considered for adoption by the state of California." (Mathematics framework (50) p.2)

Why were the hopes for the reform not realized? Again, this is a question which can be answered only by taking many factors into account. During recent years a general deterioration of the schools has been noted, and this is combined with increasing discipline problems. Effective learning of a subject such as mathematics can only be done in an appropriate atmosphere. Without this advantage, even the best curricula would be a total failure. What was regarded as one of the reform's greatest strengths, namely, the careful attention to scientific findings, proved to be one of its greatest weaknesses. As the reformers were focusing on scientific findings, they overlooked many various and often unforeseeable problems arising from field trials. Furthermore, reformers in mathematics and the representatives of other fields assumed that their own interests would interest students, so that they tended to neglect the actual performance data. They sometimes either concentrated on the social concerns or swung to the other extreme and forgot the child.

Perhaps the greatest problem in the reform was that of finding enough qualified teachers. For several decades teachers were being hired without sufficient qualifications as result of a teacher shortage. Also, some teachers, in another subject fully trained, found themselves teaching mathematics with an insufficient knowlege. These facts meant that new materials had to be "teacher proof" with regard to the content as well as methods. There were some in-service teacher training opportunities such as Summer Institutes, Academic Year Courses and Workshops. However, these programs reached only a small percentage of all the teachers responsible for teaching the new courses. These teachers themselves often had only vague ideas of the new curricula, and sometimes they simply could not convey their knowledge to their pupils in an understandable way. No reform can be successful if those who must accomplish it do not have the necessary qualifications.

It might be possible to compensate for this situation if favorable learning conditions exist such as small classes, well-equipped classrooms, adequate teacher-preparation time, sufficient quantities of instructional materials and limited discipline problems. Thus, the failure of many children who had been taught in "New Mathematics" is connected with their teachers' deficiencies.

Although the curriculum reform may not have reached its goals either entirely or only approximately, depending on one's point of view, the development of mathematical instruction had taken a long step forward. Objectives, materials and methods as well as the latest scientific and technological findings were all taken into consideration for the first time. The child came into the foreground. No longer were things simply hammerred into his head. Indeed, improvement of his attitude towards mathematics is a critical task. The student should have a strong interest in mathematics as well as enjoyment in mathematics lessons.

There are still a great many problems left for the future. Research must be pushed forward in identifying those factors which determine the learning and teaching of mathematics. The curricula need to be studied and weaknesses eliminated. Some efforts have been recently made to develop state-wide mathematics programs. Through this attempt a larger dissemination could be made possible. By using new school financing systems in which state funding plays a larger role, more of poor schools may obtain curricula materials.

Finally, there must be an attempt to bridge the gap between theory and practice. Teachers' Centers are a first attempt to accomplish this. Increased efforts in empirical testing and evaluation are necessary. But, the biggest problem seems to be pre- and in-service teacher education. This proble was the greatest obstacle to realizing the reform. Qualified teachers are the essential prerequisite.

This chapter closes with a quotation which describes the state of mathematics education as it was several years ago. It provides an interesting comparison with the observations made in this chapter.

> In all, mathematics education today is much like a healthy but not untroubled teen-ager. Its pre-teen period of enthusiastic preoccupation with the relatively clear-cut issues of content development has given way to a confused recognition of additional and much more elusive problems.
> The school mathematics community is very much aware of and deeply involved with a host of problems: the diverse needs of many subcultures in the schools; questions of teaching and learning theory; the tremenduous gap between theory and practice; problems of evaluating the merits of educational programs; the difficulties of acceptance and implementation even for programs of proven worth; pivotal issues of pre-service and in-service teacher education so vital to this implementation. But the complexity of these problems compared to the earlier questions of content and curriculum is such that they have left the mathematical education community momentarily unsettled and unsure of direction and strategy. The teen-ager has acquired a healthy awareness of the broader issues of maturity.
> Mathematics education today is the healthier because it has lost a certain simplistic view of its educational challenges and an undue certainty about its answers. It has embraced more fully problems that have always been its to address and is beginning an era of new endeavor -- charting, exploring and hoping to conquer more formidable reaches than ever before. (CBMS (17) p.149)

Epilogue

This paper should be regarded as an attempt to provide some insights into the reform movement in school mathematics in the USA beginning in 1951 and lasting for about two and a half decades. Because of the scope of the subject only certain aspects could be treated. In this paper the projects were summarized and other influencing forces were given. The projects were compared to help the reader see the main emphases of their work. This had to be done on a somewhat superficial level because of the magnitude of all the efforts of the reform.

The period of "New Math" has come to an end. It was neither a revolution of school mathematics as some proponents had foretold nor was it a disaster as some critics had proclaimed. Rather, it should be regarded as an evolution within school mathematics. The aims of the projects evolved and eventually became the following:

- to teach the kind of mathematics that pupils will find useful in their later life.
- to teach mathematics in an effective as well as enjoyable way.

To achieve these aims the following factors have to be taken into consideration:

- demands and needs of society
- new technological developments
- psychological and pedagogical findings on how children learn and how they can enjoy learning mathematics
- special needs of various student population.

There has been much complaining about decreasing test scores and many attempts to find remedies. This problem has been given too much attention. Test results are as dependent on the test difficulty and the selection of topics as they are on how much the students have learned. There is no one set of "super" objectives, but each group determines the objectives it considers appropriate.

The impression is given that the reform primarily tried to increase achievement scores. But that overlooks the many changes in content and methods which were designed to meet the new needs of today's society and to take advantage of new developments which provide improved learning and learning conditions. The demand for assessment trends to force mathematics to retreat. There are signs that some of the positive achievements of the reform are not fully appreciated because they are difficult to test and are therefore often omitted.

Another comment may be made on achievement. Striving for good test results disguises the great danger of teaching only those things that will be required in the test. Further, if the main aim becomes achieving good results there is little regard for how the necessary knowledge and skills are obtained. Such things as reaching higher levels of understanding, noticing useful relationships, experimenting so that students find out things for themselves will easily be neglected. These things are important learning experiences that will not occur if everything they must know is presented by the teacher or via programmed lessons or if they solve every assigned problem exactly as demonstrated by the teacher. Sometimes the child does not acquire an insight because he has had no chance of asking his own questions. There are no questions, only facts.

One way out of this dilemma has been shown by CSMP who uses a "pedagogy of situations".[1] That is, the child is exposed to a "pedagogically rich and meaningful situation" that continues "to stimulate intellectual involvement at all levels of mathematical sophistication".[2]

> We think that, if nothing else, the intensity of criticism of the "new math" should be forcing the educational community to reexamine its rationale for giving mathematics such an exalted place in the curriculum and to look for new and more realistic purposes for teaching mathematics. We suggest

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1) Kaufman, Haag (40) pp.287 ff.
2) Kaufman, Haag (40) p.287

> that one such rationale and purpose might be to give students some fairly good idea of what the discipline of mathematics is about. This means knowing the kinds of problems that mathematics can tackle, the kinds of methods that mathematics can apply to solve problems, the kinds of standards by which correctness in mathematical argumentation is judged. This means to know something about the important ideas in mathematics and to be intellectually involved in studying these ideas. This means, in short, to know something about our mathematical heritage. (Kaufman, Haag (40) pp.291 f.)

Another way of teaching, called the "genetic approach" by Wagenschein and Freudenthal has been used successfully for a long time in Europe. It corresponds to the inquiry methods of science education in US. It means that one must involve the child in such a way that he develops the ability to ask the questions to which he wants to find an answer.

Some ways of teaching have been given here which might have some impact on future projects in mathematics education. The main emphases on the content side will be problem solving and basic skills in the broadest sense. Much more needs to be done in defining and implementing them. Other important points for the future are the use of technological developments, further research in testing, even greater flexibility of the curriculum, improved pre- and in-service teacher education and the role of public support. Most of these aspects are covered by NCTM's Recommendations for School Mathematics of the 1980s, which may be an indication of the way mathematics might proceed in the future.[1)]

> The National Council of Teachers of Mathematics recommends that -
> 1. problem solving be the focus of school mathematics in the 1980s;
> 2. basic skills in mathematics be defined to encompass more than computational facility;
> 3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;

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1) compare recommendations pp.128 f.

4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accomodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.
(NCTM (57) p.1)

Comprehensive List of Projects[1)]

- Accelerated Instruction Project of the California Mathematics Improvement Programs
- Advanced General Mathematics and Applied Business Industrial Mathematics (AGM-III & ABI Math IV) (1967 - 1969)
- The Arithmetic Project Course for Teachers (1958 - 1973)
- Bald Rock Environcap Project (1971 - 1974)[2)]
- Bilingual Continuous Progress Mathematics (BCPM) (1971 - 1973)
- Biomedical Interdisciplinary Curriculum Project (BICP) (1968 - 1978)
- Boston College Mathematics Institute (BCMI) (1957 -)
- Calculators in General Math (1967 - 1969)
- Central Midwestern Regional Educational Laboratory (CEMREL), Inc. Comprehensive School Mathematics Program - Elementary Component (CSMP) (1967 -)
- CEMREL, Inc. Comprehensive School Mathematics Program - Elements of Mathematics (CSMP - EM) (1966 -)
- Central Iowa Low-Achiever Mathematics Project (CILAMP) (1967 -)
- Committee on Low-Achievers in Mathematics - Denver Area (COLAMDA) (1968 -)
- Committee on the Undergraduate Program in Mathematics (CUPM) (1958 -)
- Computer Assisted Diagnostic Prescriptive Program in Reading and Mathematics (CADPP) (1975 -)
- Computer Assisted Instruction Laboratory in Mathematics and Science (CAI) (1967 -)
- Computer-based Mathematics Instruction at the Stanford-Based Laboratory for Learning and Teaching (1963 -)

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1) This list is taken from J. D. Lockard (47) and (48). Some more recent programs as described in this paper are added.
2) This as well as all other interdisciplinary projects are included in this list.

- Conceptually Oriented Mathematics Program (COMP) (1971 -)
- Cooperative General Science Program (CGSP) (1966- 1971)
- Crystal Lake Outdoor Education Program (1967 -)
- Curriculum and Text-book Project (1966 - 1976)
- Dartmouth Secondary School Computer Project (1967 - 1970)
- Demonstration and Experimentation in Computer Training and Use in Secondary Schools (1967 -)
- Developing Mathematical Processes (DMP) (1967 -)
- Diagnostic Prescriptive Arithmetic (DPA) (1971 -)
- ECO - Curriculum Development & Learning Laboratory (1971 - 1974)
- Educational Research Council (ERC) Unified Science Program (1971 -)
- Elementary School Science Project (1959 - 1966)
- Elementary Science Study (ESS) (1961 - 1971)
- Engineering Concepts Curriculum Project - Man and his Technology (ECCP-MHT) (1971 -)
- Engineering Concepts Curriculum Project - The Man-Made World (ECCP-TMMW) (1965 -)
- Engineering Concepts Curriculum Project - Technology - People - Environment (ECCP-TPE) (1970 -)
- Environmental Education (1966 -)
- Environmental Education - Community Opportunity for Stewardship (ECOS) (1971 - 1974)
- The Greater Cleveland Mathematics Program of the Educational Research Council of America (GCMP-ECRCMP) (1959 -)
- ESSENTIA: Environmental Studies (ES) (1969 - 1974)
- Experience in Outdoor Education (EOE) (1966 -)
- Exploring Number Concepts (ENC) (1973 - 1975)
- Go Metrics
- Grand Rapid Mathematics Laboratory Project (1967 - 1969)
- Implementation of the New Jersey State Master Plan for Environmental Education (1971 -)
- Individual LecturesFilm Project of the Mathematical Association of America

- Individually Prescribed Mathematics Instruction (IPI Math)
- Interrelated Mathematics Science Project - I.M.S. (1968 -)
- Jefferson County Curriculum Development (1968 -)
- K-12 Multidisciplinary Process Curriculum in Environmental Education (1973 - 1974)
- Knowledge for Use Project (1973 -)
- Project LOCAL (1967 -)
- Lanakila Mathematics Project (1973 -)
- The Madison Project of Syracuse University and Webster College (1957 -)
- Math Idea Kit Series I (K-3), II (3-6) (1972 - 1973)
- Mathematics for Careers (1972 -)
- Mathematics Inquiry in the Conrad Area (MICA) (1970 - 1973)
- Mathematics/Science Education Learning System (1966 -)
- Mathematics Specialized Project of the California Mathematics Improvement Programs
- Metrics Made Easy
- Metrics - The Hawaii Metric Program (1972 - 1976)
- Minnesota Mathematics and Science Teaching Project (MINNEMAST) (1961 -)
- Mobile Environmental Education Laboratory (MEEL) (1972)
- Modern Coordinate Geometry, A Weslyan Experimental Curricular Study (1964 - 1965)
- Monona Grove, K-12 Unified Concept Centered Science Program (1964 -)
- Pattern in Arithmetic (PIA) (1964 -)
- Pennsylvania Retrieval of Information for Mathematics Education System (PRIMES) (1965 -)
- People and Technology (P & T) (1969 - 1975)
- Portland Project (1967 - 1971)
- Program for Low-Achievers in Mathematics (PLAM) (1970 - 1974
- Programmed Math Tutorial Project (PTM) (1970 - 1974)

- Project LOCAL (Laboratory Program for Computer Assisted Learning) (1967 -)
- Project for the Mathematical Development of Children (PMDC) (1973 -)
- School Mathematics Study Group (SMSG) (1963 - 1972)
- Science - A Process Approach (AAAS) (1963 - 1974)
- Secondary School Mathematics Curriculum Improvement Study (SSMCIS) (1965 - 1972)
- South Central Kansas Elementary Math-Science Project (SOCKEMS) (1969 - 1972)
- Student Centered Science Program (SCSP) (1969 -)
- Students' Instructional Gaming Network (SIGN) (1960 -)
- Success in Mathematics Through Aural Reading Techniques (SMART) (1967 - 1973)
- Systematic Teaching and Measuring Mathematics (STAMM)
- Technology for Children (1968 -)
- Total Education in the Total Environment (TETE) (1964 -)
- Unified Science and Mathematics for Elementary School (USMES) (1970 - 1978)
- University of Illinois Arithmetic Project (at Education Center) (1958 -)
- University of Illinois Committee on School Mathematics (UICSM) (1951 -)
- University of Maryland Mathematics Project (UMMaP) (1957 -)
- Upper Mississippi River ECO-Center (1972 - 1975)
- Wisconsin Mathematics Curriculum Publications

Abbreviations

AAAS - American Association for the Advancement of Science
AMS - American Mathematical Society
AT - The Arithmetic Teacher
BCMI - Boston College Mathematics Institute
CADPP - Computer Assisted Diagnostic Prescriptive Program in Reading and Mathematics
CAI - Computer Assisted Instruction
CBMS - Conference Board of the Mathematical Sciences
CCSM - Cambridge Conference on School Mathematics
CEEB - College Entrance Examination Board
CEMREL - Central Midwestern Regional Educational Laboratory
CILAMP - Central Iowa Low-Achiever Mathematics Project
COLAMDA - Committee on Low-Achievers in Mathematics - Denver Area
COMP - Conceptually Oriented Mathematics Program
CRT - Criteria Referenced Test
CSMP - Comprehensive School Mathematics Program
CTBS - Comprehensive Test of Basic Skills
DPA - Diagnostic Prescriptive Arithmetic
ERCMP - Educational Research Council Mathematics Program
ERIC - Educational Resources Information Center
ESEA - Elementary and Secondary Education Act
ETC - Educational Testing Service
GCMP - Greater Cleveland Mathematics Project
HEW - Department of Health, Education and Welfare
IEP - Individualized Educational Plan
JDRP - Joint Dissemination Review Panel
MAA - Mathematics Association of America
MPhS - Mathematisch-Physikalische Semesterberichte
MT - The Mathematics Teacher
NACOME - National Advisory Committee on Mathematics Education
NAEP - National Assessment of Educational Progress
NCSM - National Council of Supervisors of Mathematics

NCTM - National Council of Teachers of Mathematics
NDEA - National Defense Education Act
NDN - National Diffusion Network
NEA - National Education Association
NIE - National Institute of Education
NLSMA - National Longitudinal Study of Mathematical Abilities
NSF - National Science Foundation
OECD - Organization for Economic Cooperation and Development
PRISM - Priorities in School Mathematics
PSSC - Physical Science Study Committee
SMSG - School Mathematics Study Group
SSMCIS - Secondary School Mathematics Curriculum Improvement Study
STAMM - Systematic Teaching and Measuring Mathematics
UICSM - University of Illinois Committee on School Mathematics
UMMaP - University of Maryland Mathematics Project
USMES - Unified Science and Mathematics for Elementary School
USOE - United States Office of Education

Translations of German Quotations

pp.7, 8

To sum up, innovations within mathematics are characterized by

1. increased investigation and establishment of a foundation of mathematics which, with the help of mathematical logic and set theory, was highly developed as an independent discipline during the last hundred years,
2. reorganization into mathematical structures in which the classical topics have been organized according to their innate properties. This reorganization renders an overview of the whole field of mathematics and causes considerable economy of thinking;
3. development of new fields such as Boolean Algebra, computer science, statistics, etc.

p.8

Today mathematics is regarded as

being reconstructed and (in connection with different application problems) further developed by means of set theory and the axiomatic method.

p.22

The spiral approach means that concepts are dealt with anew periodically, each time on a higher level of cognition... At lower levels of cognition the student discovers mathematical concepts empirically from his environment. Later, he develops step by step analytical thinking and applies the 'axiomatic method' for representing and analyzing mathematical structures.

p.23

This means more precisely:

(1) The teaching of every subject is to be oriented primarily at the fundamental ideas ("structures") of the particular branch of science. That is, mathematics teaching is to be oriented primarily at those fundamental ideas which allow efficient description and interpretation of concepts and relationships in reality as well as within mathematics.
(2) The fundamental ideas of each subject can be taught in an appropriate manner to every child regardless of his age and social origin if the teaching is done based on the child's mental capabilities and on media which he understands.

p.24

It was one of the few projects in this phase of the curriculum reform which was guided by a behavioristic conception, which means

> it tries to set up a hierarchy of behavioristic mathematical educational objectives and to translate these into programmed instruction curricula. It attempts to relate B. S. Bloom's "Taxonomy of Educational Objectives" to mathematical aims and goals in a proper way which are then translated logically and with meaningful regard to learning theory into a sequence of measurable and specific instructional objectives.

p. 25

> Teaching means to arrange the learning conditions that exist outside the learner. These conditions are set up in stages. At each stage adequate sonsideration must be given to the learner's newly gained achievement potentials as well as the need for maintaining these potentials and setting the necessary stimulus-situation for the next stage.

p.33

- The concept of sets finds its way through all branches of mathematics in all grades; concepts and symbols of set theory are used continually and in a moderate form.
- Understanding of mathematical structures is strongly emphasized. Fundamental concepts, their properties and interrelationships build logical systems, these combine isolated facts of the contents, topics, even whole branches of mathematics.
- The concept of operation is generalized; operations with one, two or more elements are investigated; the commutative property, associative property, distributive property, identity, inverse elements and operations are investigated. Operations are not restricted to numbers.
- Students are thoroughly acquainted with different number systems, including calculation within these systems. The main reason is a better understanding of the decimal-based system. Thereby practical applications of the binary number system in calculators are not forgotten.
- Measuring is another unifying principle which appears in the entire mathematics curriculum over and over again. The student is to learn the importance of measurement units, the arbitrariness of their definition; he is supposed to learn step by step the concept of quantity; the distance between two points, the measure of an angle, the measure of other geometric elements.

- Fundamental changes have been made in algebra; it is presented as a coherent logical structure based on a relatively small number of ideas. Gradually the students learn the properties of the group, ring, and field concepts.
- Vector algebra forms an independent mathematical system; it is used for the solution of many different problems.

p.39

The aim of this project was

> to develop the whole mathematics curriculum for grades K - 12 as a comprehensive sequence of units which are in logical agreement with one another. This curriculum is to meet standards set for science development with regard to mathematical content and organization as well as requirements of a behavioristic model for setting up operationalized objectives.

p.74

> From all these efforts and setting up of projects results a real explosion in curriculum development. Altogether thirteen mathematics projects came into being between 1951 and 1963, fifteen new mathematics projects were added between 1964 until 1966, and the year 1967 alone records the setting up of twelve new projects. However, most of them are small, short-ranged projects with narrow goals.

p.88

> The concrete working objetives of USMES were following:
>
> 1. the development of 32 units together with teacher's resource books, pupils' work cards and a list of possible teaching and learning aids or other devices, information on scientific principles from integrated sciences (background papers) and proposals for the classroom management of USMES units.
> 2. the development of a resource library which provides an overview of the literature concerning USMES materials in short summaries and which is supplemented by its own descriptions of similar materials of other projects or publishers.
> 3. the development of a "design lab", that is, a laboratory equipped with various tools, instruments, devices and materials which the pupils may use to construct, to build, to measure - as part of their USMES learning and research work. It even includes computers with programmed learning units for fast acquisition of technical skills which might become necessary for problem solving processes.

p.124

These centers are

information- and communication centers for teachers - partly set up by teachers - where problems arising from daily work, curricular problems, decisions on organization and structure can be presented and discussed.

pp.124 f.

1. Redefining and establishing the basic aims of schools
2. Determination of how far the present aims have been reached by existing programs
3. Ascertaining of unmet needs
4. Development of alternative strategies for meeting these needs
5. Feedback to decision-makers of the identified priorities by which innovation in school development is possible
6. Initiation of a working process which assures that a "rolling reform" will translate theory into practice in a manner that is responsive to criticism as well as solution proposals.

p.132

The starting point of this conception is the systematizing of mathematics science based on the "structures" set up by the Bourbaki group by using a uniform language and more precise concept formulation.

p. 135

The structural conception is based on research by cognitive oriented learning theorists in concept formation on which Jerome Bruner based his theory "Structures of the Disciplines" This theory includes the following reasoning: Cognitive structures are the totality of all acquired concepts and thinking skills. They develop from simple structures consisting of only a few concepts to more and more highly developed cognitive structures as new concepts are acquired. In its most highly developed form the cognitive structures correspond to the structures of the science, the essence of all concepts and processes of which it is comprised. These structures are sufficiently complex that they embrace all basic insights, concepts and methods of the science. At the same time they are so simple to formulate that they also can be transmitted on a lower level of cognition.

p.136

The main task of curriculum construction is to find meaningful and stimulating didactic models as embodiment of structures for these decision making processes.

The functional conception starts from two assumptions:

- All school education aims at developing the pupil's optimal basic cognitive, affective and motivational abilities.
- These abilities can be described as factors of personality characteristics and as such they can be identified by psychology, socialization theory and intelligence theory.

p.137

This interdisciplinary conception

arises simultaneously and from the same cognition-theoretic basis as the functional concention. It attempts, however, to arrive at content determinations by going beyond the assertions on methodology that are made in the functional conception.

Bibliography

(1) Adler, Irving. "The Cambridge Report: Blueprint or Fantasy?" The Mathematics Teacher (MT) 1966: 210-217

(2) Beckmann, Milton W. "NDEA Support for Improved Mathematics Instruction." MT 1960: 445-447

(3) Begle, E. G. "SMSG: The First Decade." MT 1968: 239-245

(4) Bell, Frederick H. "Can Computers Really Improve School Mathematics?" MT 1978: 428-433

(5) Bell, Max S. "Calculators in Secondary School Mathematics." MT 1978: 405-411

(6) Bloom, Benjamin S. and others. "Taxonomy of Educational Objectives, Handbook 1: Cognitive Domain." David McKay Company, Inc. New York, 1956

(7) Brown, Kenneth E. "National Enrollments in High School Mathematics." MT 1956: 366-367

(8) Brown, Kenneth E., Abell, Theodore L. "Trends in Mathematical Offerings and Enrollments." MT 1966: 652-655

(9) Bruner, Jerome S. "The Process of Education." Vintage Books, New York, 1963

(10) California Mathematics Framework, Addendum, California State Department of Education, Sacramento, February 1, 1980

(11) Carpenter, Th. P., Coburn, T. G. and others. "Results and Implications of the NAEP Mathematics Assessment: Elementary School." The Arithmetic Teacher (AT) 1975: 438-450

(12) Carpenter, Th. P. and others. "Results and Implications of the NAEP Mathematics Assessment: Secondary School." MT 1975: 453-470

(13) The Center for Science and Mathematics Education at the Ohio State University. "Investigations in Mathematics Education." Columbus, Ohio

(14) Computer Assisted Diagnostic Prescriptive Program in Reading and Mathematics. Awareness Booklet. Dillwyn, Virginia

(15) Comprehensive School Mathematics Program: Basic Program Plan. CEMREL 1969

(16) Conceptually Oriented Mathematics Program. ESEA Title III. COMP consultants, Inc. Columbia, Missouri

(17) Conference Board of the Mathematical Sciences (CBMS), National Advisory Committee on Mathematics Education (NACOME). "Overview and Analysis of School Mathematics Grades K-12." Washington D.C. 1975

(18) Cress, Joe. "Modern Math." MT 1967: 139

(19) Davis, Robert D. "The Changing Curriculum: Mathematics." Association for Supervision and Curriculum Development, NEA, 1967

(20) Davis, Robert D. "The Madison Project of Syracuse University." MT 1960: 571-575

(21) Diagnostic Prescriptive Arithmetic of Staten Island ESEA Title III. New York City Board of Education, New York

(22) Dilworth, R.P., Warren, L.M. "Final Report, Specialized Teacher Project 1971/72, Mathematics Improvement Programs." San Diego, June 1973

(23) Education Committee of the Industrial Mathematics Society. "The Increasing Importance of Mathematics." MT 1953: 3-7

(24) Elbers, Doris. "Curriculumreformen in den USA. Ein Bericht ueber theoretische Ansaetze und praktische Reformverfahren mit einer Dokumentation ueber Entwicklungsprojekte." Studien und Berichte 28, Max-Planck-Institut fuer Bildungsforschung, Berlin 1973

(25) Ellrott, Schindler. "Reform des Mathematikunterrichts." Bad Heilbrunn 1975

(26) Exum, Jerry. "CSMP Today." MT 1973: 538-539

(27) Fehr, Howard F. "Reform des Mathematikunterrichts an den Secondary Schools in den Vereinigten Staaten." Mathematisch-Physikalische Semesterberichte (MPhS), Goettingen, VII/2 (1961): 224-234

(28) Fehr, Howard F. "Reform of Mathematics Education around the World." MT 1965: 37-44

(29) Fehr, Howard F. "Reorientation in Math Education." MT 1968: 593-601

(30) Fehr, Howard F. "The Secondary School Mathematics Curriculum Improvement Study: A Unified Mathematics Program." MT 1974: 25-33

(31) Fehr, Howard F. "A Unified Mathematics Program for Grades Seven through Twelve." MT 1966: 463

(32) Ferguson, W. Eugene. "Current Reforms in the Mathematics Curricula - a Passing Phase or Progress?" MT 1964: 143-148

(33) Fey, James T. "Mathematics Teaching Today: Perspectives from Three National Surveys." MT 1979: 490-504

(34) Gerbaulet, Herz, Huber u.a. "Schulnahe Curriculumentwicklung. Ein Vorschlag zur Errichtung Regionaler Paedagogischer Zentren mit Analysen ueber Innovationsprobleme in den USA, England und Schweden." Stuttgart 1972

(35) Goals for School Mathematics: The Report of the Cambridge Conference on School Mathematics. Boston 1963

(36) Greenberg, Herbert J. "The Objectives of Mathematics Education." MT 1974: 639-643

(37) Grundlach, B.H. "The Greater Cleveland Mathematics Program." AT 1961: 191-195

(38) Jelinek, Milos. "Ueber die Bestrebungen zur Modernisierung des Mathematikunterrichts." Mathematik in der Schule, 2/1965: 351-368

(39) Kansky. "COLAMDA: Activities for the Low Achiever in the Secondary School." MT 1973: 634-635

(40) Kaufman, B., Haag V. "New math or old math - The wrong question." AT 1977: 287-292

(41) Kaufman, B., Sterling N. "The CSMP Elementary School Program." CEMRECL Inc. St. Louis, Missouri, April 1978

(42) Keedy, M. L. "The University of Maryland Mathematics Project." MT 1959: 281-282

(43) Keitel, Christine. "Konzeptionen der Curriculumentwicklung im Bereich des Mathematikunterrichts - eine Darstellung der wichtigsten amerikanischen Curriculumreformprojekte." Wissenschaftliches Gutachten fuer das Institut fuer Didaktik der Mathematik an der Universitaet Bielefeld. September 1975

(44) Kline, Morris. "The Ancients versus the Moderns, a New Battle of the Books." MT 1958: 418-427

(45) Kline, Morris. "Why Johnny Can"t Add? The Failure of the New Math." New York 1973

(46) Lloyd, Daniel B. "Mathematically Trained Personnel Needed in the Government Service." MT 1951: 292-296

(47) Lockard, J. David (edt.). "Seventh Report of the International Clearinghouse on Science and Mathematics Curricular Development 1970." University of Maryland.

(48) Lockard, J. David (edt.). "The Nineth Report of the International Clearinghouse on Science and Mathematics Curricular Developments internationally 1956 - 1974." University of Maryland

(49) Mathematics Framework for California Public Schools - Kindergarten Through Grade Eight, The Second Strand Report. California State Department of Education, Sacramento 1972

(50) Mathematics Framework for California Public Schools - Kindergarten Through Grade Twelve. California State Department of Education, Sacramento 1975

(51) Matthews, Josephine J., Rahmlow, Harold F. "Individualized Mathematics the PLAN Way." MT 1970: 685-689

(52) Moss, Dan. "Metrics Made Easy." Portfolio, Huntington Beach, Ca.

(53) Mueller, Francis J. "The Revolution at Sputnik - Plus Ten." MT 1967: 696-706

(54) Mussen, Paul H., Conger, John H., Kagan, Jerome. "Child Development and Personality." Fourth edition. Harper & Row Publishers, 1974

(55) National Assessment of Educational Progress (NAEP). "Changes in Mathematical Achievement. 1973 - 78." Denver, August 1979

(56) National Council of Supervisors of Mathematics (NCSM). "Position Statement on Basic Skills." MT 1978: 147-152

(57) National Council of Teachers of Mathematics (NCTM). "An Agenda for Action. Recommendations for School Mathematics of the 1980s." Reston, Virginia 1980

(58) NCTM. "The Continuing Revolution in Mathematics." Washington D.C. 1970

(59) NCTM. "The Growth of Mathematical Ideas, Grades K-12." Twenty-Fourth Yearbook, Washington D.C. 1959

(60) NCTM. "A History in Mathematics Education in the United States and Canada." Thirty-Second Yearbook, Washington D.C. 1970

(61) NCTM."Insights into Modern Mathematics." Twenty-Third Yearbook. Washington D.C. 1957

(62) NCTM. "Position Statement on Basic Skills." MT 1978: 147

(63) NCTM. Secondary Curriculum Committee. "The Secondary Mathematics Curriculum." MT 1959: 389-417

(64) National Diffusion Network (NDN). "Educational Programs that Work." Far West Laboratory for Educational Research and Development, San Francisco, Fall 1979

(65) NDN Reporter, No. 7, March/April 1980

(66) National Institute of Education (NIE) "Conference on Basic Mathematical Skills and Learning. Volume I: Contributed Position Papers." Euclid, Ohio 1975

(67) NIE. "Conference on Basic Mathematical Skills and Learning. Volume II: Working Group Reports," Euclid, Ohio, 1975

(68) NIE/National Science Foundation (NSF). "Report of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics." Arlington, Va. 1976

(69) NSF. "Curriculum Development for Pre-College Science Education." Report prepared for the Committee on Science and Technology. U.S. House of Representatives, Ninety-Fourth Congress, Washington D.C. 1975

(70)"On the Mathematics Curriculum of the High School." MT 1962: 191-195

(71) Organization for Economic Cooperation and Development (OECD). "New Thinking in School Mathematics." Paris 1961

(72) OECD. "Mathematics Today." Paris 1963

(73) OECD. "Synopsis for Modern Secondary School Mathematics." Paris 1961

(74) Piaget, J., Inhelder, B. "The Psychology of the Child." New York 1969

(75) "Proceedings of the Conference on the Future of Mathematical Education." Tallahassee, Florida, 1975

(76) "PRISM: Shedding light on NCTM's Recommendations for the 1980s." MT 1980: 647 f.

(77) Project STAMM, Minisampler, Lakewood Co.

(78) "Report of the Conference on the K-12 Mathematics Curriculum." Snowmass, Colorado, 1973

(79) Rosenberg, Herman. "The Real Menace of the Sputniks to Mathematics Education." School Science and Mathematics 1959: 723-729

(80) Scandura, Joseph H. and others. "Federal Funds for the Improvement of Mathematics Education." AT 1965: 496-499

(81) Sharpe, Glynn. "Jefferson County Public Schools Mathematics Program." MT 1973: 635

(82) Sherman. "Common Elements in New Mathematics Programs." 1972

(83) "Status Report: Mathematics Curriculum-Development Project Today." AT 1972: 391-395

(84) Steiner, Hans-Georg. "Mathematics Curriculum Development in the USA. A Look at the Past Twenty Years." Zentralblatt fuer Didaktik der Mathematik (ZDM) Heft 3, September 1976: 136-141

(85) Steiner, Hans-Georg. "Mathematische Grundlagenstandpunkte und Reform des Mathematikunterrichts." Mathematisch-Physikalische Semesterberichte, Goettingen, XII/1 (1965): 1-22

(86) Stone, Marshall H. 'Review of "Goals for School Mathematics: The Report of the Cambridge Conference on School Mathematics." MT 1965: 353-360

(87) Taylor, Ross. "The Question of Minimum Competency as Viewed from the Schools." MT 1978: 88-93

(88) United States Office of Education (USOE). "Education in Action." (article on COMP). Washington D.C. September 1976

(89) Weaver, Fred J.: "The Cambridge Conference on School Mathematics." AT 1964: 207-210

(90) Williams, Emmet D., Shuff, Robert V. "Comparative Study of SMSG and traditional Mathematics Text Material." MT 1963: 495-504

(91) Willoughby, Stephen S. "Individualization," MT 1979: 338-344

(92) Wittmann, Erich. "Grundfragen des Mathematikunterrichts." Third edition. Braunschweig 1975

(51a)Minimum Competency - a special issue. MT 1978 p.85